IJCRR
Vol 06 issue 09
Section: General Sciences
Category: Research
Received on: 12/02/14
Revised on: 10/03/14
Accepted on: 10/04/14

THERMO-MAGNETIC CONVECTION OF AN OLDROYDIAN FLUID THROUGH A BRINKMAN POROUS MEDIUM

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ABSTRACT

Objective: The objective in the present investigation is to study the effect of magnetic field on thermal convection of an Oldroydian viscoelastic fluid through a Brinkman porous medium.
Methods: The normal mode method is used to obtain the dispersion relation.
Conclusions: For the case of stationary convection, Oldroydian viscoelastic fluid behaves like an ordinary Newtonian fluid and medium permeability and medium porosity have destabilizing effects on the system whereas the magnetic field and Darcy-Brinkman number have stabilizing effects on the system. It is also found that the modes may be oscillatory and non-oscillatory and the principle of exchange of stabilities is valid under certain condition.
Keywords: Stability, Magnetic Field, Brinkman Porous Medium.

INTRODUCTION

The problem of the onset of convection in a horizontal layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics and hydromagnetics has been discussed in detail by Chandrasekhar (1981).A porous medium is defined as a material consisting of a solid matrix with an interconnected void.A comprehensive and detailed study of thermal convection through various porous mediums has been given by Nield andBejan (2006). Tissues can be treated as a porous medium as it is composed of dispersed cells separated by connective voids which allow for flow of nutrients, minerals, etc. There are several evidences, both theoretical and experimental, which suggest that the Darcy's equation gives inadequate resultsof the hydrodynamic conditions particularly near the boundaries of a porous medium. The Darcy-Brinkman equation, which takes into account the boundary effects, has been employed in recent years in biomedical hydrodynamic studies (Khaled and Vafai, 2003).

An experimental demonstration by Toms and Strawbridge (1953) reveals that a dilute solution of methyl methacrylate in n-butyl acetate behaves in accordance with the theoretical model of Oldroyd fluid (1958).Sharma (1975) studied the stability of a layer of an electrically conducting Oldroyd fluid in the presence of a magnetic field and found that the magnetic field has a stabilizing influence.Sharma and Sunil (1994) considered the thermal instability of an Oldroydian viscoelastic fluid permeated with suspended particles in hydromagnetics in a porous medium and concluded that for the case of stationary convection, magnetic field has a stabilizing effect whereas medium permeability and suspended particles have destabilizing effects on the system. Kumar *et al.* (2013) investigated theoretically the influences of dust particles, variable gravity and magnetic field of an Oldroydian Viscoelastic fluid through a Brinkman Porous Medium.

The purpose of the present study is to investigate the problem of the onset of stability of an Oldroydian viscoelastic fluid through a Brinkman porous medium in hydromagnetics.

RESEARCH METHODOLOGY

The following Research Methodology is adopted for the proposed Research paper:

- Identification of the problem
- Collection and study of related literature
- Mathematical formulation of the problem
- Stability analysis and use of normal mode method
- Interpretation of results
- Conclusion

FORMULATION OF THE PROBLEM

Consider an infinite horizontal layer of an Oldroydian viscoelastic fluid bounded by the planes z=0 and z=d in a porous medium of porosity $\in$ and medium permeability k_1. The fluid layer is acted on by a uniform vertical magnetic field **H** (0, 0, H). The governing equations of motion and continuity for anOldroydian viscoelastic fluid are defined as:

$$\frac{\rho_0}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\in}(q.\nabla)q\right]=\left(1+\lambda\frac{\partial}{\partial t}\right)\begin{bmatrix}-\nabla p+\rho_0\mathbf{X_i}+\\ \frac{\mu_e}{4\pi}(\nabla\times H)\times H\end{bmatrix} + \left(1+\lambda_0\frac{\partial}{\partial t}\right)\left[\frac{\tilde{\mu}\nabla^2}{\in}-\frac{\mu}{k_1}\right]q \quad (1)$$

$$\nabla.q=0 \quad (2)$$

Where in the above equations, $\rho_0, \mu, \tilde{\mu}, \mu_e, q and\ \mathbf{X_i}$ denote, respectively, the density of fluid, viscosity, effective viscosity, magnetic permeability, velocity of pure fluid and the gravitational acceleration term.

The energy equation is defined as:

$$\left[\in\rho c_v+\rho_s c_s(1-\in)\right]\frac{\partial T}{\partial t}+\rho c_v(q.\nabla)T=k\nabla^2 T \quad (3)$$

Where $\rho_s, c_s, c_v, T\ and\ k$ denote, respectively, the density of solid material, heat capacity of solid material, the specific heat at constant volume, the temperature and the thermal conductivity.

The Maxwell's equation yields

$$\in\frac{\partial H}{\partial t}=\nabla\times(q\times H)+\in\eta\nabla^2 H \quad (4)$$

and $\nabla.H=0,$ (5)

Where η denote the electrical resistivity.

The equation of state is

$$\rho=\rho_0\left[1+\alpha(T_0-T)\right] \quad (6)$$

The steady state solution corresponding to the system of equations (1)-(6) are defined as:

$$q=(0,0,0),\ T=T_0-\beta z,\ \rho=\rho_0[1+\alpha\beta z],$$
$$p=p_0-g\rho_0\left[1+{\alpha\beta z^2}/{2}\right], H=[0,0,H] \quad (7)$$

Let the initial state solutions described by equations (7) be slightly perturbed. We assume that **q** (u,v,w), θ, $\delta p, \delta\rho$ and $h\left(h_x, h_y, h_z\right)$ denote, respectively, the perturbation in fluid velocity **q**(0,0,0), temperature T, pressure p, density ρ and magnetic field H.The change in density $\delta\rho$ caused by perturbation θ in temperature, is given by

$$\delta\rho = -\alpha\theta\rho_0 \qquad (8)$$

The governing linearized perturbation equations are defined as:

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\begin{bmatrix} -\frac{1}{\rho_0}\nabla\left(\delta p\right) + g\alpha\theta\lambda_i + \\ \frac{\mu_e}{4\pi\rho_0}\left\{\left(\nabla\times h\right)\times H\right\} \end{bmatrix} + \left(1+\lambda_0\frac{\partial}{\partial t}\right)\left[\frac{\tilde{\upsilon}\nabla^2}{\in} - \frac{\upsilon}{k_1}\right]q \qquad (9)$$

$$\nabla . q = 0 \qquad (10)$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta \qquad (11)$$

$$\nabla . h = 0 \qquad (12)$$

$$\in\frac{\partial h}{\partial t} = \left(\nabla H\right)q + \in\eta\nabla^2 h \qquad (13)$$

where $\upsilon = \frac{\mu}{\rho_0},\ \tilde{\upsilon} = \frac{\tilde{\mu}}{\rho_0},\ \kappa = \frac{k}{\rho_0 C_v},\ \lambda_i = \left(0,0,1\right)$ *and* w denote, respectively, the kinematic viscosity, the effective kinematic viscosity, the co-efficient of thermometric conductivity, unit vertical vector and fluid velocity and $E = \in + \left(1-\in\right)\left(\frac{\rho_s C_s}{\rho_0 C_v}\right)$

Eliminating δp among the equations (9)- (13) andconsidering a solution of the form:

$$\left[w,\theta,\zeta,h_z,\xi\right] = \left[W\left(z\right),\Theta\left(z\right),Z\left(z\right),K\left(z\right),X\left(z\right)\right] \exp\left(ik_x x + ik_y y + nt\right) \qquad (14)$$

Where k_x, k_y are the wave numbers along x and y directions, respectively and $k^2 = \left(k_x{}^2 + k_y{}^2\right)$ is the resultant wave number and n is the frequency of the harmonic disturbanceand also making the substitutions of the non-dimensional quantities of the form $z = z^* d,\ a = kd,\ \sigma = \frac{nd^2}{\upsilon},\ \tau_1 = \frac{m\upsilon}{K'd^2}, \mathrm{F}_1 = \frac{\lambda_0\upsilon}{d^2}, \mathrm{F}_2 = \frac{\lambda\upsilon}{d^2}$.We obtain the non-dimensional form of the equations (9)-(13) (after dropping the asterisk for convenience) as:

$$\left[\frac{\sigma}{\in} - \frac{\left(1+F_1\sigma\right)}{\left(1+F_2\sigma\right)}\left\{\frac{D_A\left(D^2-a^2\right)}{P_l\in} - \frac{1}{P_l}\right\}\right]\left(D^2-a^2\right) W\left(z\right) + \frac{g\alpha a^2 d^2\Theta}{\upsilon} - \frac{\mu_e H d}{4\pi\rho_0\upsilon}\left(D^2-a^2\right)DK = 0 \qquad (15)$$

$$\left[\frac{\sigma}{\in}-\frac{\left(1+F_1\sigma\right)}{\left(1+F_2\sigma\right)}\left\{\frac{D_A\left(D^2-a^2\right)}{P_l\in}-\frac{1}{P_l}\right\}\right]Z=\frac{\mu_e Hd}{4\pi\rho_0\upsilon}DX \quad (16)$$

$$\left[\left(D^2-a^2\right)-Ep_1\sigma\right]\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W \quad (17)$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in K=\left(\frac{Hd}{\eta}\right)DW \quad (18)$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in X=\left(\frac{Hd}{\eta}\right)DZ \quad (19)$$

Where $D_A=\frac{\tilde{\mu}k_1}{\mu d^2}$, is the Darcy-Brinkman number, $P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1=\frac{\upsilon}{\kappa}$, is the thermal Prandtl number and $p_2=\frac{\upsilon}{\eta}$, is the magnetic Prandtl number, $\zeta=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ and $\xi=\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}$ stand for the z-component of vorticity and z-component of current density, respectively. $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the three dimensional Laplacian operator.

Eliminating Θ *and* K from equations (15)- (19), we obtain:

$$\begin{aligned}&\left[\frac{\sigma}{\in}-\frac{\left(1+F_1\sigma\right)}{\left(1+F_2\sigma\right)}\left\{\frac{D_A\left(D^2-a^2\right)}{P_l\in}-\frac{1}{P_l}\right\}\right]\\&\left[\left(D^2-a^2\right)-Ep_1\sigma\right]\left[p_2\sigma-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)\in\\&W\left(z\right)-Q\left[\left(D^2-a^2\right)-Ep_1\sigma\right]\left(D^2-a^2\right)D^2W\left(z\right)\\&=Ra^2\left[p_2\sigma-\left(D^2-a^2\right)\right]\in W\left(z\right)\end{aligned} \quad (20)$$

Where, in the above equation (20), $R=\frac{g_0\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number and $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0\upsilon\eta}$, is the Chandrasekhar number.

The boundary conditions are defined as:

$W = D^2W = DZ = DK = h_z = \Theta = 0$ on z = 0 and 1. (21)

Equation (20) together with the boundary condition (21) constitutes an eigen-value problem for the present problem. It is evident that when $F_1=0$ the system reduces to a Maxwell fluid whereas for $F_1=0$ *and* $F_2=0$, the system reduces to that for an ordinary viscous fluid.

LINEAR STABILITY ANALYSIS AND SOLUTION OF THE EIGEN-VALUE PROBLEM

Following the boundary conditions (21), a proper solution for W belonging to the lowest mode can be defined as:

$$W=W_0\sin\pi z \quad (22)$$

Where W_0 is a constant.

Substituting equation (22) in equation (20) and assuming the followings:

$R_1 = \frac{R}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $P = \pi^2 P_l$, $Q_1 = \frac{Q}{\pi^2}$, $D_A = \frac{D_{A_1}}{\pi^2}$, we get the eigenvalue relationship of the form:

$$\left[\frac{i\sigma_1}{\in} + \frac{\left(1+i\sigma_1\pi^2 F_1\right)}{\left(1+i\sigma_1\pi^2 F_2\right)}\left\{\frac{D_{A_1}\left(1+x\right)}{P\in} + \frac{1}{P}\right\}\right]$$

$$\left[\left(1+x\right)+i\sigma_1 Ep_1\right]\left[i\sigma_1 p_2 + \left(1+x\right)\right]\left(1+x\right)\in + Q_1\left(1+x\right) \quad (23)$$

$$\left[\left(1+x\right)+i\sigma_1 Ep_1\right] = R_1 x\left[i\sigma_1 p_2 + \left(1+x\right)\right]\in$$

Equation (23) is required dispersion relation including the effects of magnetic field and medium permeability on thermal instability of an Oldroydvisco-elastic fluid through a Brinkman porous medium.

THE STATIONARY CONVECTION

For the case of stationary convection, the marginal state will be characterized by substituting $\sigma = 0$ in equation (23) and obtain the eigenvalue relationshipof the form:

$$R_1 = \frac{1}{x}\left[\left\{\frac{D_{A_1}\left(1+x\right)^3}{P\in} + \frac{\left(1+x\right)^2}{P}\right\} + \frac{Q_1}{\in}\left(1+x\right)\right] \qquad (24)$$

Equation (24) shows that for the case of stationary convection Oldroydianvisco-elastic fluid behaves like an ordinary Newtonian fluid.

Minimizing equation (24) with respect to x, givesa third order equation in x of the form $4D_{A_1}x^3 + 3\left(3D_{A_1} + \in\right)x^2 + \left(9D_{A_1} + 4\in\right)x + \left(D_{A_1} + \in - Q_1 P\right) = 0$ (25)

By putting the value of critical wave number x_c obtained from equation (25), in equation (24), we can obtain the values of critical Rayleigh number for the case of stationary instability.

To investigate the effects of various parameters like magnetic field, medium permeability, Darcy-Brinkman number and medium porosity, we examine the behaviour of $\frac{dR_1}{dQ_1}, \frac{dR_1}{dP}, \frac{dR_1}{dD_{A_1}}$ *and* $\frac{dR_1}{d\in}$ analytically.

Equation (24) yields

$$\frac{dR_1}{dQ_1} = \frac{\left(1+x\right)}{\in x} \qquad (26)$$

$$\frac{dR_1}{dP} = -\frac{1}{x}\left[\frac{\left(1+x\right)^2}{P^2} + \frac{D_{A_1}\left(1+x\right)^3}{P^2\in}\right] \qquad (27)$$

$$\frac{dR_1}{dD_{A_1}} = \frac{\left(1+x\right)^3}{xP\in} \qquad (28)$$

$$\frac{dR_1}{d\in} = -\frac{1}{x\in^2}\left[\frac{D_{A_1}\left(1+x\right)^3}{P} + Q_1\left(1+x\right)\right] \qquad (29)$$

PRINCIPAL OF EXCHANGE OF STABILITIES AND OSCILLATORY MODES

In this section, we will obtain the conditions under which the principle of exchange of stabilities is valid and the possibility of oscillatory modes, if any, for Oldroydian viscoelastic fluid under the effect of magnetic field through a Brinkman porous medium.

To do this, we multiply equation (15) by W^* (the complex conjugate of W), integrating over the range of z and making use of equations (17) and (18) with the help of boundary conditions (21), we obtain

$$\left[\frac{\sigma}{\in}+\frac{(1+F_1\sigma)\in}{(1+F_2\sigma)P_l}\right]I_1-\frac{(1+F_1\sigma)}{(1+F_2\sigma)P_l}$$

$$\{D_A I_2\}-\frac{g\alpha a^2\kappa}{\beta\upsilon}\left(I_3+Ep_1\sigma^* I_4\right) \qquad (30)$$

$$+\frac{\mu_e\in}{4\pi\rho_0 p_2}\left(p_2\sigma^* I_5+I_6\right)=0$$

Where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,\ I_2=\int_0^1\left(|D^2W|^2+a^4|W|^2+2a^2|DW|^2\right)dz,\ I_3=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_4=\int_0^1\left(|\Theta|^2\right)dz,\ I_5=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,\ I_6=\int_0^1\left(|D^2K|^2+a^4|K|^2+2a^2|DK|^2\right)dz$$

All the above integrals I_1-I_6 are positive definite.

Putting $\sigma=i\sigma_0$, where σ_0 is real, into equation (30), and equating the imaginary part, we obtain

$$\sigma_0\left\{\begin{array}{l}\left(\frac{1}{\in}\right)I_1+\frac{(F_2-F_1)}{\left(1+F_2{}^2\sigma_0{}^2\right)P_l}\left(I_1\in+D_A I_2\right)\\ \frac{g\alpha a^2\kappa}{\beta\upsilon}-\left(\frac{\mu_e\in I_5}{4\pi\rho_0}\right)\end{array}\right\}=0 \qquad (31)$$

Equation (31) implies that $\sigma_0=0$ or $\sigma_0\neq 0$ which indicates that the modes may be non oscillatory or oscillatory. The oscillatory modes are introduced due to presence of magnetic field, visco-elasticity, medium permeability and Darcy-Brinkman parameter.

In the absence of magnetic field, equation (31) reduces to

$$\sigma_0\left\{\left(\frac{1}{\in}\right)I_1+\frac{(F_2-F_1)}{\left(1+F_2{}^2\sigma_0{}^2\right)P_l}\left(I_1\in+D_A I_2\right)\right\}=0 \qquad (32)$$

It is evident from equation (32) that if $F_2>F_1$ then the term inside the bracket is positive which implies that $\sigma_0=0$, thus the modes are non-oscillatory and the principle of exchange of stabilities is satisfied.

CONCLUSION

The thermal convection problem of an Oldroydian viscoelastic fluid in the presence of vertical magnetic field is considered through a Brinkman porous medium. For the case of stationary convection, it is found that the Oldroydian fluid behaves like an ordinary Newtonian fluid and the effects of medium porosity and medium permeability is to hasten the onset of thermal convection whereas magnetic field and Darcy-

Brinkman number have a stabilizing effect on the system because their effect is to postpone the onset of thermal instability.

The oscillatory modes are introduced due to presence of magnetic field, visco-elasticity, medium permeability and Darcy-Brinkman parameter. The principle of exchange of stabilities is found to hold under certain conditions. The present investigation would be relevant in the study of some stability problems of polymer solutions and Oldroydian viscoelastic fluids.

ACKNOWLEDGEMENT

Authors acknowledge theimmense help received from the scholars whose articles are cited and included inreferences of this manuscript. The authors are also grateful to authors / editors / publishers of all those articles, journals and books from where the literature for this article has been reviewed and discussed.

REFERENCES

1. Chandrasekhar, S.C. (1981). Hydrodynamic and Hydromagnetic Stability, Dover Publication, New York.
2. Khaled, A.R.A. and Vafai, K. (2003). The role of porous media in modeling flow and heat transfer in biological tissues. Int. J. Heat Mass Transfer, Vol.46, pp.4989-5003.
3. Kumar, K., Singh, V. and Sharma, S. (2013). Stability of an Oldroydian Viscoelastic Fluid Permeated with Suspended Particles through a Brinkman Porous Medium with variable gravity field in Hydromagnetics, American Journal of fluid Dynamics, Vol. 3, 3, pp. 58-66. doi: 10.5923/j.ajfd.20130303.02
4. Nield, D.A. and Bejan, A. (2006). Convection in porous media. Springer, New-York.
5. Oldroyd J.G. (1958). Non-Newtonian effects in steady motion of some idealized elastic-viscous liquids, Proc. Royal Soc. London**,** A 245, pp. 278-297.
6. Sharma R.C. (1975). Thermal instability in a viscoelastic fluid in hydromagnetics, Acta Phys. Hung., Vol. 38, pp. 293-298.
7. Sharma R.C. and Sunil. (1994). Thermal instability of Oldroydian viscoelastic fluid with suspended particles in hydromagnetics in porous medium, Polymer Plastics Technology and Engineering, Vol. 33, Issue 3, pp. 323-339.
8. Toms B.A. and Strawbridge D.J. (1953). Elastic and viscous properties of dilute solutions of polymethyl methacrylate in organic liquids, Trans. Faraday Soc., Vol. 49, pp. 1225-1232.