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PROPAGATION OF THE SHOCK WAVES IN THE EXPONENTIALLY DECREASING ATMOSPHERE

Rajan Singh[1], Mukesh Chandra[2], Anand kumar[3]

[1,2]Department of Mathematics, I.F.T.M. University, Moradabad, 244001, India; [3]Ramabai Govt. Girls P.G. College Akbarpur, Ambedkarnagar (U.P.).

ABSTRACT

The purpose of the present paper is to study propagation of the shock waves and the flow parameters in the exponentially decreasing atmosphere. Plane shock waves as it propagates vertically upward in the static atmosphere of the earth are also studied. The variation of Mach number and the velocity of the shock waves are obtained analytically by using Whitham's Rule. It is found that shock velocity increases as the shock wave propagates in the atmosphere with decreasing density normal to plane of the shock front

Key Words: Shock wave, Pressure, Density, Velocity, Mach number, Propagation, Adiabatic and isothermal atmosphere

1. INTRODUCTION

Bhatnagar and Sachdev [1] have obtained a relation between the density, pressure and Mach number by using Whitham's Rule to the propagation of the shock waves in the non-homogeneous medium. Kopal and Carrus [2], Hardy and Grover [3] have been studied the problem of propagation of the shock waves in a non-uniform medium with various density distributions and found the behavior of the fluid flow in the presence of the shock waves. Hayes [4] investigated the vorticity jump across a gas dynamics discontinuity by considering the radiative heat transfer into account and concluded that for an optically thin gas, vorticity is unaffected in pseudo-stationary flows. Kanwal [5, 6] employed the theory of generalized functions and differential geometry to study the propagation and deformation of wave front in stationary three dimensional gas flows. With minor changes the experimental data for temperature distribution given by Mitra [8,9] has been used .So the aim of the present paper is to see how the shock velocity varies in the absence of any body force as the shock waves propagate in the atmosphere, in which the density is decreasing exponentially, Whitham's Rule [10] is applied to find and approximate analytic relation for the Mach number and the shock velocity.

In this paper, we have studied the problem of propagation of the shock waves in the earth's atmosphere and also used the Whitham's rule. The experimental data for temperature distribution given by Mitra [8] has been used for minor changes.

RESEARCH METHODOLOGY

The following Research Methodology is adopted for the proposed Research paper:

- Identification of the problem
- Collection and study of related literature
- Mathematical formulation of the problem
- Analysis and numerical solution of the mathematical model
- Interpretation of results
- Conclusion

2. FORMULATION OF THE PROBLEM

Let us assume that a gaseous medium, in which the pressure, the temperature and the density vary along a fixed direction. Let us consider that the X -axis of the medium be taken along this direction and the distance measured along this axis be denoted by x. So that the thermo dynamical parameters. In the direction normal to the X -axis are constant. Let the pressure P_0 the temperature T_0 and the density p_0 at a distance x measured from a fixed plane denoted by x = 0 and P_s, T_s and

Corresponding Author:
Rajan Singh, Department of Mathematics, I.F.T.M. University, Moradabad, 244001, India.
E-mail: rajan0779@rediffmail.com

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p_s be the values of the corresponding parameters at x = 0. Then it is considered that the density and the temperature in the medium are

$$\rho_O = \rho_S e^{-\alpha_1 x} \tag{2.1}$$

$$T_O = T_S e^{-\alpha_2 x} \tag{2.2}$$

Where x is the distance measured normal to the plane of variations of the density and temperature. The subscript's' represents the value of the parameters P_0... and T_0 at a distance x =0 and α_1, α_2 are constants. It is considered that the plane shock waves start at a point x = 0 and propagates in the x-direction.

Let us consider that the medium obeying the law of gas

$$p_0 = R\rho_0 T_0 \tag{2.3}$$

Where R is the gas constant and Po is the gas pressure at a distance x from the Surface x = 0 measured normal to it. Using the equations (2.1) and (2.2) into the equation (2.3), we obtain

$$P_O = R\rho_S T_S e^{-(\alpha_1+\alpha_2)x} \tag{2.4}$$

Wherein

$$P_S = R\rho_S T_S \tag{2.5}$$

$$U = \frac{\sqrt{\gamma} U_0}{c_S} \tag{2.6}$$

Let us consider that U_0 be the shock velocity, u_0 be the fluid velocity, c_0 be the sound velocity and R_o be the shock radius. Then the non-dimensional fluid parameters are given by the following relations:

$$u = \frac{\sqrt{\gamma} u_0}{c_S} \tag{2.7}$$

$$p = \frac{p_0}{p_S} \tag{2.8}$$

$$p = \frac{p_0}{p_S} \tag{2.9}$$

$$\rho = \frac{\rho_0}{\rho_S} \tag{2.10}$$

And the dimensionless quantities r, R and the time t as

$$r = r_0 \alpha_1 \tag{2.11}$$

$$R = R_0 \alpha_1 \tag{2.12}$$

$$t = \frac{c_S t_0 \alpha_1}{\sqrt{\gamma}} \tag{2.13}$$

$$c_S{}^2 = \frac{\gamma p_S}{\rho_S} \tag{2.14}$$

The equations of motion in terms of dimensionless parameters are given by

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + u\frac{\partial}{\partial r}\left(\frac{p}{\rho^\gamma}\right) = 0 \tag{2.15}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} = 0 \tag{2.16}$$

$$p + \rho u^2 = p_1 + \rho_1 (u_1 - U)^2 \tag{2.17}$$

Here it is considered that there is no body force acting on the medium. Let a plane shock wave, created at the plane r = 0, propagates in the direction of r increasing.

It is considered that the fluid velocity in front of the shock surface is zero, when it reaches the distance r = R.

The jump conditions in the fluid parameters are

$$p + \rho u^2 = p_1 + \rho_1 (u_1 - U)^2 \tag{2.18}$$

$$p + \rho u^2 = p_1 + \rho_1 (u_1 - U)^2 \tag{2.19}$$

$$\frac{\gamma p}{\gamma - 1\rho} + \frac{1}{2}U^2 = \frac{\gamma p_1}{\gamma - 1\rho_1} + \frac{1}{2}(u_1 - U)^2 \tag{2.20}$$

2.1 For velocity u_1:

Substituting the equation (2.18) into the equation (2.19), we obtain

$$p_1 = p + \rho U u_1 \tag{2.21}$$

Further, substituting the equation (2.18) into the equation (2.20), we get

$$(U-u_1)\left[\frac{1}{2}(U-u_1)+\frac{\gamma}{\gamma-1}\frac{p_1}{\rho U}\right]=\frac{\gamma}{\gamma-1}\frac{p_1}{\rho}+\frac{1}{2}U^2 \tag{2.22}$$

Using the equation (2.21), the relation $c^2=\gamma\frac{p}{\rho}$ and U=M c into

the equation, we obtain

$$u_1(R,t)=\frac{2c}{\gamma+1}\left[M-\frac{1}{M}\right] \tag{2.23}$$

2.2. For pressure p_1:

Further: substituting U = Mc and $c^2=\gamma\frac{p}{\rho}$ into the equation (2.21), we obtain

$$p_1 = p + \rho M c u_1 \tag{2.4}$$

Putting the value of u1 into the equation (2.24), we obtain

$$p_1(R,t)=\frac{\gamma p}{\gamma+1}f(M) \tag{2.25}$$

$$f(M)=\left[2M^2-\left(1-\frac{1}{\gamma}\right)\right] \tag{2.26}$$

2.3. For density ρ_1:

Inserting the equation (2.23) and U = Mc into the equation (2.18), we obtain

$$p_1(R,t)=\frac{(\gamma+1)\rho M^2}{g(m)} \tag{2.27}$$

$$g(M)=\left[(\gamma+1)M^2+2\right] \tag{2.28}$$

$$c2=\gamma\frac{p}{\rho} \tag{2.29}$$

$$M=\frac{U}{c} \tag{2.30}$$

The relations (2.23), (2.25) and (2.27) are Rankine-Hugoniot jump conditions.

3. SOLUTION OF THE PROBLEM:

The aim of this chapter is to find how the shock behaves, when it is propagates from a plane r = 0 to r = ∞. For this it is necessary to know, how the Mach number M varies in the medium. This can know from the relations (2.23), (2.25) and (2.27), we have one extra relation connecting the four unknown variables u_1, p_1, ρ_1 and M in terms of p and ρ. This extra relation is provided by Whitham's Rule [13]. The equation of motion along the positive characteristic axis -

$$\frac{dR}{dt}=u_1+c_1$$

Are given by

$$dp_1+\rho_1 c_1 du_1=0 \tag{3.1}$$

On differentiating the equations (2.23) and (2.25), we obtain the positive characteristic axis -

$$du_1=\frac{c}{(\gamma+1)}\left[\frac{dp}{p}-\frac{d\rho}{\rho}\right]\left(M-\frac{1}{M}\right)+\frac{2c}{(r+1)}\left(\frac{M^2+1}{M^2}\right)dM \tag{3.2}$$

$$dp=\frac{p\gamma}{(\gamma+1)}\left[4M^2\frac{dM}{M}+f(M)\frac{dp}{p}\right] \tag{3.3}$$

and differentiating the equation (2.29), we get

$$2\frac{dc}{c}=\frac{dp}{p}-\frac{d\rho}{\rho} \tag{3.4}$$

Substituting the equation (3.4) into the equation (3.2), we obtain

$$du_1=\frac{c}{(\gamma+1)}\left(2\frac{dc}{c}\right)\left(M-\frac{1}{M}\right)+\frac{2c}{(r+1)}\left(\frac{M^2+1}{M^2}\right)dMa \tag{3.5}$$

The equation (2.29) can be written as

$$c_1 = \sqrt{\gamma \frac{p_1}{\rho_1}} \tag{3.6}$$

Substituting the values of p_1 and ρ_1 into the equation (3.6), we obtain

$$c_1 = \frac{c}{M(\gamma+1)}\sqrt{\gamma f(M) g(M)} \tag{3.7}$$

Further, making use of the relations (2.27), (3.3), (3.5) and (3.7) in (3.1), we obtain

$$2\left[(M^2+1)h(M)+2M^2\right]\frac{dM}{M}+f(M)\frac{dp}{p}+2(M^2-1)h(M)\frac{dc}{c}=0 \tag{3.8}$$

Wherein

$$h(M)=\sqrt{\frac{\gamma f(M)}{g(M)}} \tag{3.9}$$

Hence the equation (3.8) gives the relation between M, p and c.

The law of variation of the sound velocity and the pressure at a direction

r = R in non-dimensional parameters is

$$c(R) = e^{\frac{-\eta}{2}R} \tag{3.10}$$

$$p(R) = e^{-(\eta+1)R} \tag{3.11}$$

Where

$$\eta = \frac{\alpha_2}{\alpha_1}$$

Inserting the relations (3. 10) and (3. 11) into the relation (3.8), we obtain

$$\frac{2}{M}\frac{dM}{dR} = K(M) \tag{3.12}$$

Wherein

$$M^2 = e^{K(M)R} \tag{3.13}$$

Integrating the equation (3.12), we obtain

$$M^2 = e^{K(M)R}$$

Or

$$M = e^{\frac{1}{2}K(M)R} \tag{3.14}$$

We shall now discuss the following two cases:

Case 1: <u>Isothermal Case:</u> The equation (3.13) becomes, if we take η=0 and γ= 1 then

$$K(M) = \frac{f(M)}{\left[2M^2+(M^2+1)h(M)\right]} \tag{3.15}$$

If we take η= 1 and γ = 1 then the equations (2.26), (2.28) and (3.9) become

$$f(M) = 2M^2, g(M) = 2 \text{ and } h(M) = M \tag{3.16}$$

Using the relation (3.16) into the equation (3.15), we obtain

$$K(M) = \frac{2M}{(M+1)^2} \tag{3.17}$$

<u>Case II</u>: <u>General Case:</u> (a) If we take η= 1 and γ = 4/3 then the equation (3.13) reduces in the form

$$K(M) = \frac{f(M)+(M^2-1)h(M)}{\left[2M^2+(M^2+1)h(M)\right]} \tag{3.18}$$

we take η= 1 and γ = 4/3 ,then the equations (2.26), (2.28) and (3.9) become

$$f(M) = \frac{8M^2-1}{4}, g(M) = \frac{6+M^2}{3} \text{ and } h(M) = \sqrt{\frac{8M^2-1}{M^2+6}} \tag{3.19}$$

Further, making use of the relation (3.19) in (3.18), we obtain

$$K(M) = \frac{\frac{8M^2-1}{2}+(M^2-1)\sqrt{\frac{8M^2-1}{M^2+6}}}{\left[2M^2+(M^2+1)\sqrt{\frac{8M^2-1}{M^2+6}}\right]} \tag{3.20}$$

(b) if we take η= 1 and = 7/5 ,then the equations (2.26), (2.28) and (3.9) become

$$f(M) = 2\left[M^2 - \frac{1}{7}\right], g(M) = 2\left[1+\frac{M^2}{5}\right] \text{ and } h(M) = \sqrt{\frac{7M^2-1}{M^2+5}} \tag{3.21}$$

Substituting the equations (3.21) into the equation (3.18), we obtain

$$K(M)=\frac{\frac{4}{7}\left[7M^2-1\right]+(M^2-1)\sqrt{\frac{7M^2-1}{M^2+5}}}{\left[2M^2+(M^2+1)\sqrt{\frac{7M^2-1}{M^2+5}}\right]} \tag{3.22}$$

In the table (2.1), the function K(M) versus M is given for η =0 and γ=1 and η = 1, γ = 7/5 and $\geq$ = 4/3.

It is seen that the variation in K(M) as M $\geq$ 2 is small for η =1 and γ=1, K(M) $\to 0$ as $M \to \infty$ for the former case. We take the variation of K(M) as compared to the variations of M negligible. But for η= 0, K(M) varies from 0.4444 to 0 as M goes from 2 to ∞. Thus, if we take K(M) constant for small variation in M. The error involved is significant. The problem is considered in two sections, first a general case η = 1, η = 4/3, 7/5 and the second as isothermal case i.e. η = 0, γ =1.

Table 2.1: Variation of K(M) versus M for various values of γ and η:

Mach Number	η =0	η = 1	
M	γ =1	γ = 7/5	γ = 4/3
1.0	0.5000	0.8570	0.8754
1.5	0.4800	1. 1176	1.1231
2.0	0.4444	1.2380	1.2365
3.0	0.3750	1.3386	1.3302
4.0	0.3200	1.3775	1.3658
5.0	0.2778	1.3961	1.3827
6.0	0.2449	1.4065	1.3920
7.0	0.2188	1.4128	1.3976
8.0	0.1975	1.4169	1.4013
9.0	0.1800	1.4197	1.4038
10.0	0.1653	1.4218	1.4056
11.0	0.1528	1.4232	1.4070
12.0	0.1420	1.4244	1.4079
13.0	0.1327	1.4253	1.4088
14.0	0.1244	1.4260	1.4094
15.0	0.1171	1.4266	1.4099
16.0	0.1107	1.4270	1.4103
17.0	0.1049	1.4274	1.4106
18.0	0.0997	1.4273	1.4109
19.0	0.0950	1.4280	1.4112
20.0	0.0907	1.4283	1.4114
25.0	0.0739	1.4291	1.4121
30.0	0.0624	1.4295	1.4124
35.0	0.0540	1.4297	1.4127
40.0	0.0475	1.4299	1.4128
45.0	0.0425	1.4300	1.4129
50.0	0.0384	1.4301	1.4130
60.0	0.0322	1.4302	1.4131
70.0	0.0277	1.4303	1.4131
80.0	0.0243	1.4303	1.4 132
90.0	0.0217	1.4101	1.4112
100.0	0.0196	1.4303	1.4132
200.0	0.0099	1.4305	1.4133
500.0	0.00398	1.4305	1.4133
1000.0	0.00200	1.4305	1.4133
2000.0	0.00090	1.4305	1.4133
5000.0	0.00040	1.4305	1.4133
7000.0	0.00029	1.4305	1.4133
7500.0	0.00026	1.4305	1.4133
8000.0	0.00024	1.4305	1.4133

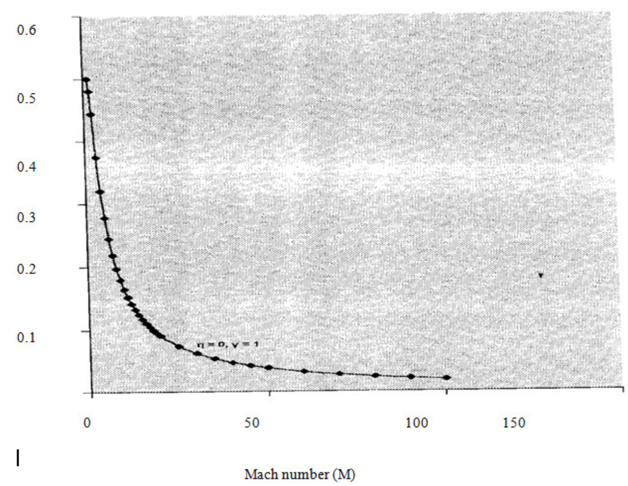

Figure 2.1

4. GENERAL CASE:

Let us assume that the variation of K(M) is small as compared to M, then the equation (3.12) reduces in the form

$$dR=\frac{2}{M}\frac{dM}{K(M)}$$

Integrating the equation (4.1) becomes

$$R=\frac{2}{K(M)}\int_{M_s}^{M}\frac{dM}{M}$$

$$R = \frac{2}{K(M)} \log \frac{M}{M_s}$$

Where M_s is the value of M at R=0.

Equation (4.2) can be written as

$$M = M_s e^{\frac{K(M)R}{2}}$$

Substituting the equations (3.10) and (4.3) into the equation (2.30), we obtain the shock velocity

$$U = M_S e^{-\left[\eta - K(M)\frac{R}{2}\right]} \quad (4.4)$$

Remark 4.1: It is noteworthy that the expressions (4.3) and (4.4) hold only for $\eta = 1$, since the variation in K(M) is not small for $\eta = 0$.

5. ISOTHERMAL CASE:

The variation in K(M) is not small for $\eta = 0$ and $\rightarrow$=1 and also K(M) $\rightarrow$0 as M$\rightarrow \infty$. It is not possible to apply the above method in this case. But it is easily seen that K(M) is very simplified for $\eta = 0$, λ=1, then

$$K(M) = \frac{2M}{(M+1)^2} \quad (5.1)$$

Using equation (5.1), then the equation (3.12) becomes

$$\frac{dM}{dR} = \left(\frac{M}{M+1}\right)^2 \quad (5.2)$$

Integrating (5.2), between the limit R=0 to R and M=M_s to M. we get

$$M^2 e^{\frac{M^2-1}{M}} = Ae^R \quad (5.3)$$

Wherein

$$A = M_S^2 e^{\frac{M_S^2-1}{M_S}} \quad (5.4)$$

The expression for the dimensionless sound velocity c is

$$c = e^{-\left(\frac{\eta}{2}\right)r} \quad (5.5)$$

Remark 5.1: It is to be noted that 11 must be zero for isothermal case. Hence equation (5.5) reduces in the form c = I and the equation (2.30) reduces in the form

$$M=U \quad (1.6)$$

This manifests that the Mach number is equal to the shock velocity. By virtue of equations (5.3) and (5 (»). we obtain all exponential for shock velocity -

$$U^2 e^{\frac{U^2-1}{U}} = Ae^R \quad (5.7)$$

In the fig.-2.1 and fig.-2.2, the shock velocity U has been drawn against the work position R for η= 1, γ=1, γ = 4/3, 7/5 and η = 0, γ =1.It is easily seen that the acceleration of the shock front is small for the isothermal case compared to that of the general case and the strength of the shock increases in the medium.

CONCLUSION

In this paper, if is very useful discussed the Plane shock waves as it propagates vertically upward in the static atmosphere of the earth studied. It is found that shock velocity increases as the shock wave propagates in the atmosphere with decreasing density normal to plane of the shock front.

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