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Static and Quasi-Static Deformation of a Uniform Half-Space Due to a Center of Rotation

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ABSTRACT

Objective: The objective is to obtain static and quasi-static deformation of a uniform half-space due to a center of rotation.

Methodology and Results: The Galerkin vector approach has been used to calculate deformation field at an arbitrary point of an elastic half-space. Closed form analytical expressions for the displacements and stresses caused by a center of rotation buried in a homogenous, isotropic, perfectly elastic half-space are derived. The quasi-static deformation field for a viscoelastic medium has been obtained by applying the correspondence principle of linear viscoelasticity to the associated elastic solution. Explicit expressions giving the quasi-static deformation of a uniform half-space caused by a center of rotation are obtained when the medium is elastic in dilatation and Kelvin, Maxwell or SLS type viscoelastic in distortion.

Conclusion: The explicit expressions for the displacements and stresses in an elastic and viscoelastic medium due to a center of rotation source have been obtained. Numerical results are shown graphically for displacements and stresses.

Key Words: Center of rotation, Static and quasi-static deformation, Correspondence principle, Viscoelastic, Maxwell

INTRODUCTION

Nuclei of strain are the concentrated sources of a displacement field and are built up from the simple superposition of single forces which are acting at a point in the elastic medium (Love[1]). Therefore, the displacement field owing to nuclei of strain is necessary for applications to crustal deformation. Analytical expressions for three-dimensional static displacement fields owing to a step-type single force in an elastic half-space, have been obtained by Mindlin[2] by using Galerkin's method. Mindlin and Cheng[3] derived the solutions in the form of the Galerkin vector for various nuclei of strain in a uniform half-space by the process of superposition, differentiation and integration. Many theoretical formulations have been developed (Okada[4]), which were describing the deformation of an isotropic, homogeneous, and semi-infinite medium. Analytical expressions for the quasi-static surface displacements due to a vertical strike-slip fault in a Kelvin or Maxwell viscoelastic half-space were determined by Singh and Rosenman[5] by applying the correspondence principle of linear viscoelasticity. The correspondence principle has been broadly used to calculate the quasi-static deformation of a viscoelastic half-space by a point or extended sources (see. e.g. [6-10]). Singh and Singh[11] have identified the combinations in which moduli occur in the expressions for the displacements, strains and stresses in a uniform elastic half-space due to buried sources. Using analytical integration, the displacement field in two welded elastic half-spaces due to a finite rectangular fault has been obtained by Singh et al[12] and they have also compared it with the corresponding field in an elastic half-space and in an infinite medium. To model the ground deformation in volcanic areas, Singh et al[13] used four axially-symmetric source models in an elastic half-space and also compared it with the corresponding field due to a center of dilatation.

Cochard et al[14] showed that the observations of seismic rotational motions will give important new information referring to the Earth's surface and are complementary to those found from the observations of the translational motions of the Earth's surface using conventional seismometers. Cowsik et al[15] resolved that to detect rotational motions, the basic design concept of using a torsion balance as a filter is validated and can be implemented for the construction of rotational seismometers.

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The static displacements from the seismic recordings and identifying translation signals caused by rotation can be estimated by using rotational motions (Trifunac and Todorovska[16]). Rotational seismology is of great interest to a wide range of geophysical disciplines, including strong-motion seismology, seismic tectonics, earthquake engineering, and geodesy as well as to physicists using Earth-based observatories for detecting gravitational waves generated by astronomical sources (Lee et al[17]). The solutions for the displacement field produced by a center of rotation is also useful for many purposes and in bio-mechanical research.

RESEARCH METHODLOGY

In this paper, we study the 3-D deformation of a uniform half–space caused by a center of rotation by using the Galerkin vector approach. Explicit expressions for the static strains can be easily obtained with the help of strain -displacement relations and the stresses follow immediately by using stress-strain relations. The correspondence principle of linear viscoelasticity has been used to obtain the quasi-static displacements, strains and stresses.

The paper has been divided into two parts. Part-A deals with the static deformation field while the quasi-static deformation field is considered in Part-B.

PART- A: STATIC DEFORMATION FIELD

THE GALERKIN VECTOR

Mindlin and Cheng[3] expressed the displacement vector $\vec{u}$ in term of the Galerkin vector $\vec{G}$ through the relation

$$2\mu\vec{u} = 2(1-\sigma)\nabla^2\vec{G} - \nabla(\nabla.\vec{G}) \tag{2.1}$$

where μ is the shear modulus and σ is the Poisson's ratio. From equation (2.1), the displacement u_i in the x_i - direction is given by

$$u_i = \frac{1}{2\mu}\left[2(1-\sigma)\nabla^2 G_i - \frac{\partial}{\partial x_i}(\nabla.\vec{G})\right] \tag{2.2}$$

where $\vec{G} = G_i\,\vec{e}_i$ and $\vec{e}_i$ denotes the unit vector in the x_i – direction.

The strain-displacement relations are

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2.3}$$

where e_{ij} is the strain tensor.

The stress-strain relations are

$$\tau_{ij} = \lambda\delta_{ij}u_{n,n} + 2\mu e_{ij} \tag{2.4}$$

where τ_{ij} is the stress-tensor, δ_{ij} is the Kronecker delta and λ,μ are Lame's constants. From equation (2.1) and (2.3),

$$e_{nn} = u_{n,n} = div\vec{u} = \frac{1-2\sigma}{2\mu}\nabla^2(\nabla.\vec{G}) \tag{2.5}$$

The expressions for the Galerkin vectors for a vertical and horizontal concentrated force in a homogeneous, isotropic, elastic half-space were given by Mindlin and Cheng≥. Let the uniform half-space be z ≥0 with a stress-free boundary at z=0 and the point source is located at the point (0,0, c) of the upper half space as shown in Fig.1. R_1 is the distance between the observer at (x, y, z) and the source at (0,0, c); R_2 is the distance between the observer at (x, y, z) and the image at (0,0, - c),

where

$$R_1^2 = x^2 + y^2 + (z-c)^2$$

$$R_2^2 = x^2 + y^2 + (z+c)^2 \tag{2.6}$$

CENTER OF ROTATION

Combining two coplanar single couples with moments equal in magnitude and sense, we get a torque or center of rotation, in which the net force is zero, whereas the net moment is non-zero, as shown in Fig.2.

For further study, we use (x, y, z) co-ordinate instead of (x_1, x_2, F_o).

Taking strength of center of rotation along x-axis, y-axis and z-axis as F_o which acts at the point $(0,0,c)$, the expressions for Galerkin vectors are given as under:

Center of rotation about x-axis

$$\vec{G} = \frac{F_o}{8\pi(1-\sigma)}\left[\vec{e}_2\left\{-\frac{z-c}{R_1} + \frac{z-3c}{R_2} + \frac{2c^2(z+c)}{R_2^3} + 4(1-\sigma)(1-2\sigma)\log(R_2+z+c)\right\} + \vec{e}_3 y\left\{\frac{1}{R_1} + \frac{4\sigma(1-2\sigma)+3}{R_2} - \frac{2c^2}{R_2^3} + \frac{4(1-2\sigma)[(1-\sigma)z - c\sigma]}{R_2(R_2+z+c)}\right\}\right] \tag{2.7}$$

Center of rotation about y-axis

$$\vec{G}=\frac{F_o}{8\pi(1-\sigma)}\left[\begin{array}{l}\vec{e}_1\left\{\begin{array}{l}-\frac{z-c}{R_1}+\frac{z-3c}{R_2}+\frac{2c^2(z+c)}{R_2^3}+\\ 4(1-\sigma)(1-2\sigma)\log(R_2+z+c)\end{array}\right\}+\\ \vec{e}_3x\left\{\begin{array}{l}\frac{1}{R_1}+\frac{4\sigma(1-2\sigma)+3}{R_2}-\frac{2c^2}{R_2^3}+\\ \frac{4(1-2\sigma)[(1-\sigma)z-\sigma c]}{R_2(R_2+z+c)}\end{array}\right\}\end{array}\right] \quad (2.8)$$

Center of rotation about z-axis

$$\vec{G}=\frac{F_o}{8\pi(1-\sigma)}\left[(\vec{e}_2x-\vec{e}_1y)\left\{\begin{array}{l}\frac{1}{R_1}+\frac{1}{R_2}+\frac{2c^2}{R_2^3}\\ -\frac{4(1-\sigma)(1-2\sigma)}{R_2+z+c}\end{array}\right\}\right] \quad (2.9)$$

DISPLACEMENT FIELD

Center of rotation about x-axis

Using equations (2.2) and (2.7), we obtain the following expressions for the displacement components for a center of rotation in the x - direction

$$u_1^{(1)}=\frac{F_oxy}{4\pi\mu}\left[\frac{2(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2}-\frac{6z}{R_2^5}\right] \quad (3.1)$$

$$u_2^{(1)}=\frac{F_o}{4\pi\mu}\left[\begin{array}{l}\frac{z-c}{R_1^3}+\frac{z-c}{R_2^3}-\frac{2(1-2\sigma)}{R_2(R_2+z+c)}\\ +y^2\left(\frac{2(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2}-\frac{6z}{R_2^5}\right)\end{array}\right] \quad (3.2)$$

$$u_3^{(1)}=\frac{F_oy}{4\pi\mu}\left[-\frac{1}{R_1^3}-\frac{6z(z+c)}{R_2^5}-\frac{3-4\sigma}{R_2^3}\right] \quad (3.3)$$

Center of rotation about y-axis

Using equations (2.2) and (2.8), we obtain the following expressions for the displacement components for a center of rotation in the y- direction

$$u_1^{(2)}=\frac{F_o}{4\pi\mu}\left[\begin{array}{l}\frac{z-c}{R_1^3}+\frac{z-c}{R_2^3}-\frac{2(1-2\sigma)}{R_2(R_2+z+c)}\\ +x^2\left(\frac{2(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2}-\frac{6z}{R_2^5}\right)\end{array}\right] \quad (3.4)$$

$$u_2^{(2)}=\frac{F_oxy}{4\pi\mu}\left[\frac{2(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2}-\frac{6z}{R_2^5}\right] \quad (3.5)$$

$$u_3^{(3)}=\frac{F_ox}{4\pi\mu}\left[-\frac{1}{R_1^3}-\frac{6z(z+c)}{R_2^5}-\frac{3-4\sigma}{R_2^3}\right] \quad (3.6)$$

Center of rotation about z-axis

Using equations (2.2) and (2.9), we obtain the following expressions for the displacement components for a center of rotation in the z– direction

$$u_1^{(3)}=\frac{F_oy}{4\pi\mu}\left[\frac{1}{R_1^3}+\frac{1}{R_2^3}\right] \quad (3.7)$$

$$u_2^{(3)}=\frac{F_ox}{4\pi\mu}\left[-\frac{1}{R_1^3}-\frac{1}{R_2^3}\right] \quad (3.8)$$

$$u_3^{(3)}=0 \quad (3.9)$$

STRAIN AND STRESS FIELD

The strain components e_{ij} can be easily obtained by using the displacements components u_i as expressed in previous section in relation (2.3) and the stresses follow from equation (2.4), which are given as below:

STRESSES

Stress components due to center of rotation about x-axis

$$\tau_{11}^{(1)}=\frac{F_0y}{2\pi}\left\{\begin{array}{l}\frac{6c}{R_2^5}-\frac{6(1-2\sigma)(z+c)}{R_2^5}+\frac{2(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\\ +x^2\left[\begin{array}{l}-\frac{4(1-2\sigma)(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3}-\frac{2(1-2\sigma)(z+c)}{R_2^5(R_2+z+c)^2}\\ +\frac{30z}{R_2^7}\end{array}\right]\end{array}\right\} \quad (4.1)$$

$$\tau_{22}^{(1)}=\frac{F_0y}{2\pi}\left\{\begin{array}{l}-\frac{3(z-c)}{R_1^5}-\frac{3(5z-c-4\sigma(z+c))}{R_2^5}\\ +\frac{6(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2}+\\ y^2\left[\begin{array}{l}-\frac{4(1-2\sigma)(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3}-\frac{2(1-2\sigma)(z+c)}{R_2^5(R_2+z+c)^2}\\ +\frac{30z}{R_2^7}\end{array}\right]\end{array}\right\} \quad (4.2)$$

$$\tau_{33}^{(1)}=\frac{F_0y}{2\pi}\left\{\frac{3(z-c)}{R_1^5}-\frac{3(z-c)}{R_2^5}+\frac{30z(z+c)^2}{R_2^7}\right\} \quad (4.3)$$

$$\tau_{12}^{(1)} = \frac{F_0 x}{4\pi}\left\{\begin{array}{l} -\frac{3(z-c)}{R_1^5} - \frac{3(3z-c)}{R_2^5} + \frac{4(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} \\ +y^2\left[\begin{array}{l} \frac{60z}{R_2^7} - \frac{4(1-2\sigma)(z+c)}{R_2^5(R_2+z+c)^2} \\ -\frac{8(1-2\sigma)(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} \end{array}\right] \end{array}\right\} \tag{4.4}$$

$$\tau_{23}^{(1)} = \frac{3F_0}{4\pi}\left\{\begin{array}{l} \frac{-(z-c)^2}{R_1^5} - \frac{(3z^2+2zc-c^2)}{R_2^5} \\ +y^2\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right) \end{array}\right\} \tag{4.5}$$

$$\tau_{31}^{(1)} = \frac{3F_o xy}{4\pi}\left\{\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right)\right\} \tag{4.6}$$

Stress components due to center of rotation about y-axis

$$\tau_{11}^{(2)} = \frac{F_0 x}{2\pi}\left\{\begin{array}{l} -\frac{3(z-c)}{R_1^5} - \frac{3(5z-c-4\sigma(z+c))}{R_2^5} \\ +\frac{6(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} + \\ x^2\left[\begin{array}{l} -\frac{4(1-2\sigma)(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} - \frac{2(1-2\sigma)(z+c)}{R_2^5(R_2+z+c)^2} \\ +\frac{30z}{R_2^7} \end{array}\right] \end{array}\right\} \tag{4.7}$$

$$\tau_{22}^{(2)} = \frac{F_0 x}{2\pi}\left\{\begin{array}{l} \frac{6c}{R_2^5} - \frac{6(1-2\sigma)(z+c)}{R_2^5} + \frac{2(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} \\ +y^2\left[\begin{array}{l} -\frac{4(1-2\sigma)(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} - \frac{2(1-2\sigma)(z+c)}{R_2^5(R_2+z+c)^2} \\ +\frac{30z}{R_2^7} \end{array}\right] \end{array}\right\} \tag{4.8}$$

$$\tau_{33}^{(2)} = \frac{F_0 x}{2\pi}\left\{\frac{3(z-c)}{R_1^5} - \frac{3(z-c)}{R_2^5} + \frac{30z(z+c)^2}{R_2^7}\right\} \tag{4.9}$$

$$\tau_{12}^{(2)} = \frac{F_0 y}{4\pi}\left\{\begin{array}{l} -\frac{3(z-c)}{R_1^5} - \frac{3(3z-c)}{R_2^5} + \frac{4(1-2\sigma)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} \\ +x^2\left[\begin{array}{l} \frac{60x_3}{R_2^7} - \frac{4(1-2\sigma)(z+c)}{R_2^5(R_2+z+c)^2} \\ -\frac{8(1-2\sigma)(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} \end{array}\right] \end{array}\right\} \tag{4.10}$$

$$\tau_{23}^{(2)} = \frac{3F_o xy}{4\pi}\left\{\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right)\right\} \tag{4.11}$$

$$\tau_{31}^{(2)} = \frac{3F_0}{4\pi}\left\{\begin{array}{l} \frac{-(z-c)^2}{R_1^5} - \frac{(3z^2+2zc-c^2)}{R_2^5} \\ +x^2\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right) \end{array}\right\} \tag{4.12}$$

Stress components due to center of rotation about z-axis

$$\tau_{11}^{(3)} = \frac{3F_o xy}{2\pi}\left\{\left(-\frac{1}{R_1^5} - \frac{1}{R_2^5}\right)\right\} \tag{4.13}$$

$$\tau_{22}^{(3)} = \frac{3F_o xy}{2\pi}\left\{\left(\frac{1}{R_1^5} + \frac{1}{R_2^5}\right)\right\} \tag{4.14}$$

$$\tau_{33}^{(3)} = 0 \tag{4.15}$$

$$\tau_{12}^{(3)} = \frac{3F_o}{4\pi}\left\{(x^2-y^2)\left(\frac{1}{R_1^5} + \frac{1}{R_2^5}\right)\right\} \tag{4.16}$$

$$\tau_{23}^{(3)} = \frac{3F_o x}{4\pi}\left\{\left(\frac{z-c}{R_1^5} + \frac{z+c}{R_2^5}\right)\right\} \tag{4.17}$$

$$\tau_{31}^{(3)} = \frac{3F_o x}{8\pi\mu}\left\{\left(-\frac{z-c}{R_1^5} - \frac{z+c}{R_2^5}\right)\right\} \tag{4.18}$$

PART B. QUASI-STATIC DEFORMATION

BASIC EQUATIONS

The stress-strain relations (2.4) for an isotropic, linear elastic medium, may be expressed in the form

$$p_{ij} = 2\mu\varepsilon_{ij};\theta = 3\kappa\vartheta \tag{5.1}$$

where

$$\theta = \tau_{11} + \tau_{22} + \tau_{33}$$

$$\vartheta = e_{11} + e_{22} + e_{33}$$

p_{ij} = deviatoric stress $= \tau_{ij} - \frac{1}{3}\theta\delta_{ij}$,

ε_{ij} = deviatoric strain $= e_{ij} - \frac{1}{3}\vartheta\delta_{ij}$, and

$\kappa = \lambda + \frac{2}{3}\mu$ is known as bulk modulus

The stress-strain relations for an isotropic, linear viscoelastic medium, may be expressed in the form

$$\bar{p}_{ij}(s) = 2\bar{\mu}(s)\bar{\varepsilon}_{ij}(s), \bar{\theta}(s) = 3\bar{\kappa}(s)\bar{\vartheta}(s) \quad (5.2)$$

where

$$\bar{\mu}(s) = \frac{\bar{Q}_1(s)}{2\bar{P}_1(s)}, \bar{\kappa}(s) = \frac{\bar{Q}_2(s)}{3\bar{P}_2(s)}$$

where are the polynomials of degree m_1, m_2, n_1, n_2 (positive integers), respectively.

From equations (5.1) and (5.2), we note that on replacing the elastic moduli μ and κ by the transformed moduli $\bar{\mu}$ and $\bar{\kappa}$ respectively, the Laplace transformed viscoelastic solution can be obtained from the corresponding transformed elastic solution. This is the well-known correspondence principle of viscoelasticity.

The elastic moduli μ and κ occur in various combinations in the expressions for the displacements, strains, and stresses. For these combinations, correspondence between the static solution for an elastic medium and quasi-static solution in the time domain for a viscoelastic medium was given in table-1(Singh and Singh[10]). We can easily obtain the quasi-static response of a viscoelastic half-space to various sources from the static responses given in Part-1 by using table-1.

The quasi-static displacements, strains and stresses for center of rotation are obtained when the medium is elastic in dilatation and kelvin, Maxwell or SLS type viscoelastic in distortion.

QUASI-STATIC DISPLACEMENTS

Quasi-static displacements due to center of rotation about x-axis

$$u_1^{(1)} = \frac{F_o xy}{4\pi}\left[\frac{6\hat{Q}_3(t)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} - \frac{12\hat{J}_1(t)z}{R_2^5}\right] \quad (5.3)$$

$$u_2^{(1)} = \frac{F_o}{4\pi}\left[\begin{array}{l} 2\hat{J}_1(t)\left(\frac{z-c}{R_1^3} + \frac{z-c}{R_2^3} - \frac{6zy^2}{R_2^5}\right) \\ -6\hat{Q}_3(t)\left(\frac{1}{R_2(R_2+z+c)} - \frac{y^2(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\right)\end{array}\right] \quad (5.4)$$

$$u_3^{(1)} = \frac{-F_o y}{4\pi}\left[\begin{array}{l} 2\hat{J}_1(t)\left(\frac{1}{R_1^3} + \frac{6z(z+c)}{R_2^5} + \frac{1}{R_2^3}\right) \\ + \frac{6\hat{Q}_3(t)}{R_2^3}\end{array}\right] \quad (5.5)$$

Quasi-static displacements due to center of rotation about y-axis

$$u_1^{(2)} = \frac{F_o}{4\pi}\left[\begin{array}{l} 2\hat{J}_1(t)\left(\frac{z-c}{R_1^3} + \frac{z-c}{R_2^3} - \frac{6zx^2}{R_2^5}\right) \\ -6\hat{Q}_3(t)\left(\frac{1}{R_2(R_2+z+c)} - \frac{x^2(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\right)\end{array}\right] \quad (5.6)$$

$$u_2^{(2)} = \frac{F_o xy}{4\pi}\left[\frac{6\hat{Q}_3(t)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} - \frac{12\hat{J}_1(t)z}{R_2^5}\right] \quad (5.7)$$

$$u_3^{(2)} = \frac{-F_o x}{4\pi}\left[\begin{array}{l} 2\hat{J}_1(t)\left(\frac{1}{R_1^3} + \frac{6z(z+c)}{R_2^5} + \frac{1}{R_2^3}\right) \\ + \frac{6\hat{Q}_3(t)}{R_2^3}\end{array}\right] \quad (5.8)$$

Quasi-static displacements due to center of rotation about z-axis

$$u_1^{(3)} = \frac{F_o y}{4\pi}\left[2\hat{J}_1(t)\left(\frac{1}{R_1^3} + \frac{1}{R_2^3}\right)\right] \quad (5.9)$$

$$u_2^{(3)} = \frac{-F_o x}{4\pi}\left[2\hat{J}_1(t)\left(\frac{1}{R_1^3} + \frac{1}{R_2^3}\right)\right] \quad (5.10)$$

$$u_3^{(3)} = 0 \quad (5.11)$$

QUASI-STATIC STRESSES

Quasi-static stresses due to center of rotation about x-axis

$$\tau_{11}^{(1)} = \frac{F_0 y}{2\pi}\left\{\begin{array}{l} \frac{6c}{R_2^5} - 3\hat{Q}_4(t)\left(\frac{3(z+c)}{R_2^5} - \frac{(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\right) \\ +x^2\left[\frac{30z}{R_2^7} - 3\hat{Q}_4(t)\left(\begin{array}{l}\frac{2(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} \\ +\frac{z+c}{R_2^5(R_2+z+c)^2}\end{array}\right)\right]\end{array}\right\} \quad (5.12)$$

$$\tau_{22}^{(1)} = \frac{F_0 y}{2\pi}\left\{ -\frac{3(z-c)}{R_1^5} - \frac{9(z-c)}{R_2^5} - 9\hat{Q}_4(t)\left(\frac{z+c}{R_2^5} - \frac{(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\right) + y^2\left[\frac{30z}{R_2^7} - 3\hat{Q}_4(t)\left(\frac{2(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} + \frac{z+c}{R_2^5(R_2+z+c)^2}\right)\right]\right\} \quad (5.13)$$

$$\tau_{33}^{(1)} = \frac{F_0 y}{2\pi}\left\{\frac{3(z-c)}{R_1^5} - \frac{3(z-c)}{R_2^5} + \frac{30z(z+c)^2}{R_2^7}\right\} \quad (5.14)$$

$$\tau_{12}^{(1)} = \frac{F_0 x}{4\pi}\left\{ -\frac{3(z-c)}{R_1^5} - \frac{3(3z-c)}{R_2^5} + \frac{6\hat{Q}_4(t)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} + y^2\left[\frac{60z}{R_2^7} - 6\hat{Q}_4(t)\left(\frac{z+c}{R_2^5(R_2+z+c)^2} + \frac{2(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3}\right)\right]\right\} \quad (5.15)$$

$$\tau_{23}^{(1)} = \frac{3F_0}{4\pi}\left\{ \frac{-(z-c)^2}{R_1^5} - \frac{(3z^2+2zc-c^2)}{R_2^5} + y^2\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right)\right\} \quad (5.16)$$

$$\tau_{31}^{(1)} = \frac{3F_o xy}{4\pi}\left\{\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right)\right\} \quad (5.17)$$

Quasi-static stresses due to center of rotation about y-axis

$$\tau_{11}^{(2)} = \frac{F_0 x}{2\pi}\left\{ -\frac{3(z-c)}{R_1^5} - \frac{9(z-c)}{R_2^5} - 9\hat{Q}_4(t)\left(\frac{z+c}{R_2^5} - \frac{(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\right) + y^2\left[\frac{30z}{R_2^7} - 3\hat{Q}_4(t)\left(\frac{2(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} + \frac{z+c}{R_2^5(R_2+z+c)^2}\right)\right]\right\} \quad (5.18)$$

$$\tau_{22}^{(2)} = \frac{F_0 x}{2\pi}\left\{ \frac{6c}{R_2^5} - 3\hat{Q}_4(t)\left(\frac{3(z+c)}{R_2^5} - \frac{(2R_2+z+c)}{R_2^3(R_2+z+c)^2}\right) + y^2\left[\frac{30z}{R_2^7} - 3\hat{Q}_4(t)\left(\frac{2(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3} + \frac{z+c}{R_2^5(R_2+z+c)^2}\right)\right]\right\} \quad (5.19)$$

$$\tau_{33}^{(2)} = \frac{F_0 x}{2\pi}\left\{\frac{3(z-c)}{R_1^5} - \frac{3(z-c)}{R_2^5} + \frac{30z(z+c)^2}{R_2^7}\right\} \quad (5.20)$$

$$\tau_{12}^{(2)} = \frac{F_0 y}{4\pi}\left\{ -\frac{3(z-c)}{R_1^5} - \frac{3(3z-c)}{R_2^5} + \frac{6\hat{Q}_4(t)(2R_2+z+c)}{R_2^3(R_2+z+c)^2} + x^2\left[\frac{60z}{R_2^7} - 6\hat{Q}_4(t)\left(\frac{z+c}{R_2^5(R_2+z+c)^2} + \frac{2(2R_2+z+c)^2}{R_2^5(R_2+z+c)^3}\right)\right]\right\} \quad (5.21)$$

$$\tau_{23}^{(2)} = \frac{3F_o xy}{4\pi}\left\{\left(\frac{20z(z+c)}{R_2^7} + \frac{1}{R_1^5} - \frac{1}{R_2^5}\right)\right\} \quad (5.22)$$

$$\tau_{31}^{(2)}=\frac{3F_0}{4\pi}\left\{\begin{matrix}\frac{-(z-c)^2}{R_1^5}-\frac{(3z^2+2zc-c^2)}{R_2^5}\\ +x^2\left(\frac{20z(z+c)}{R_2^7}+\frac{1}{R_1^5}-\frac{1}{R_2^5}\right)\end{matrix}\right\} \qquad (5.23)$$

Quasi-static stresses due to center of rotation about z-axis are same as static expressions of stresses given in section 4.1.3.

NUMERICAL RESULTS

We define dimensionless epicentral distance (D), dimensionless horizontal displacement (U), dimensionless vertical displacement (W), dimensionless stresses (τ_x,τ_z) by the relations $D=\frac{x}{c}, U=\frac{A}{c}u_2^{(1)}, W=\frac{A}{c}u_2^{(3)}$, $\tau_x = A\ \tau_{12}^{(1)}$ and $\tau_z = A\ \tau_{12}^{(3)}$ where $A=\frac{-2\pi\mu c^3}{F_0}$ is dimensionless constant.

For further calculations, the following assumptions are considered: the half-space $z \geq 0$ is stress-free at z=0, the bulk modulus κ and the shear modulus μ of viscoelastic half-space satisfy Poisson's condition $\frac{\kappa}{\mu}=\frac{5}{3}$. We also assume that t>0 and the source time function is the Heaviside step function. Using the values of various auxiliary functions are given by Singh and Singh11, considering the material as an elastic in dilatation and Kelvin, Maxwell, or SLS viscoelastic in distortion.

STATIC DISPLACEMENTS

From equations (3.1) -(3.9), the dimensionless static displacements are as follows:

$$U=\frac{1}{(D^2+1)^{\frac{1}{2}}}\left[\frac{1}{(D^2+1)}+\frac{(1-2\sigma)}{\left((D^2+1)^{\frac{1}{2}}+1\right)}\right] \qquad (6.1)$$

$$W=\frac{D}{(D^2+1)^{\frac{3}{2}}} \qquad (6.2)$$

QUASI-STATIC DISPLACEMENTS

From equations (5.3) -(5.11), the dimensionless Quasi-static displacements for various viscoelastic materials is obtained as under:

Kelvin model

$$U=\frac{1}{(D^2+1)^{\frac{1}{2}}}\left[\frac{(1-e^{-T})}{(D^2+1)}+\frac{(1-e^{-6T})}{2\left((D^2+1)^{\frac{1}{2}}+1\right)}\right] \qquad (6.3)$$

$$W=\frac{D(1-e(-T))}{(D^2+1)^{\frac{3}{2}}} \qquad (6.4)$$

where $T=\frac{t}{t_1}$ and t_1 is relaxation time for Kelvin model.

Maxwell model

$$U=\frac{1}{(D^2+1)^{\frac{1}{2}}}\left[\frac{1+T}{(D^2+1)}+\frac{3}{5}\frac{\left(1-\frac{1}{6}e^{\frac{-5}{6}T}\right)}{\left((D^2+1)^{\frac{1}{2}}+1\right)}\right] \qquad (6.5)$$

$$W=\frac{D(1+T)}{(D^2+1)^{\frac{3}{2}}} \qquad (6.6)$$

where $T=\frac{t}{t_2}$ and t_2 is relaxation time for this model.

SLS model

$$U=\frac{2}{(D^2+1)^{\frac{1}{2}}}\left[\frac{\left(1-\frac{1}{2}e^{\frac{-1}{2}T}\right)}{(D^2+1)}+\frac{3}{11}\frac{\left(1-\frac{1}{12}e^{\frac{-11}{12}T}\right)}{\left((D^2+1)^{\frac{1}{2}}+1\right)}\right] \qquad (6.7)$$

$$W=\frac{2D\left(1-\frac{1}{2}e^{\frac{-1}{2}T}\right)}{(D^2+1)^{\frac{3}{2}}} \qquad (6.8)$$

where $T=\frac{t}{t_2}$ and t_2 is relaxation time for SLS model.

STATIC STRESSES

From equations (4.1) -(4.18), the dimensionless static stresses are as follows:

$$\tau_x=\frac{-D}{(D^2+1)^{\frac{3}{2}}}\left[\frac{3}{(D^2+1)}+\frac{2(1-2\sigma)\left(2(D^2+1)^{\frac{1}{2}}+1\right)}{\left((D^2+1)^{\frac{1}{2}}+1\right)^2}\right] \qquad (6.9)$$

$$\tau_z=\frac{-3D^2}{(D^2+1)^{\frac{5}{2}}} \qquad (6.10)$$

QUASI-STATIC STRESSES

From equations (5.12) -(5.17), the dimensionless quasi-static stresses for various viscoelastic models are as follows:

Kelvin model

$$\tau_x=\frac{-3D}{(D^2+1)^{\frac{3}{2}}}\left[\frac{1}{(D^2+1)}+\frac{\left(2(D^2+1)^{\frac{1}{2}}+1\right)(1+5e^{-6T})}{3\left((D^2+1)^{\frac{1}{2}}+1\right)^2}\right] \qquad (6.11)$$

Maxwell model

$$\tau_x=\frac{-3D}{(D^2+1)^{\frac{3}{2}}}\left[\frac{1}{(D^2+1)}+\frac{\left(2(D^2+1)^{\frac{1}{2}}+1\right)e^{\frac{-5}{6}T}}{3\left((D^2+1)^{\frac{1}{2}}+1\right)^2}\right] \qquad (6.12)$$

SLS model

$$\tau_x = \frac{-3D}{\left(D^2+1\right)^{\frac{3}{2}}}\left[\frac{1}{\left(D^2+1\right)} + \frac{2\left(2\left(D^2+1\right)^{\frac{1}{2}}+1\right)\left(1+\frac{5}{6}e^{\frac{-11}{12}T}\right)}{11\left(\left(D^2+1\right)^{\frac{1}{2}}+1\right)^2}\right] \quad (6.13)$$

DISCUSSION

Fig. 3. shows the effect of the Poisson's ratio on the variation of the dimensionless horizontal displacement (U) with the dimensionless epicentral distance (D). We note that, horizontal displacement increases with the decrease of Poisson's ratio and epicentral distance. It vanishes at D=1.272 for $\sigma = 1$.

Fig. 4. shows the variation of the dimensionless vertical displacement (W) with the dimensionless epicentral distance. We note that, it increases with epicentral distance and assumes maximum value W=0.35 at D=1 after that it decreases rapidly as epicentral distance increases.

Fig. 5. shows the variation of the dimensionless stress τ_x with the dimensionless epicentral distance for various values of σ. We note that, it increases with the increase of Poisson's ratio and assumes minimum value -0.8 at σ=1/2 for D= 0.6 after that it increases with epicentral distance and becomes constant as D tends to infinity.

Fig. 6. shows the variation of the dimensionless stress τ_z with the dimensionless epicentral distance. We note that, stress τ_z decreases with epicentral distance and assumes minimum value at D=1 after that it increases with epicentral distance.

Fig. 7. shows the variation of horizontal displacement for the three models namely Kelvin, Maxwell and SLS with the epicentral distance for different values of T. We note that in case of Kelvin model, horizontal displacement decreases with the epicentral distance as shown in Fig.7(a) but in case of Maxwell model(Fig.7(b)) and SLS model(Fig.7(c)), it decreases first and then increases with epicentral distance.

Fig. 8 shows the variation of the vertical displacement for the three models namely Kelvin, Maxwell and SLS with the epicentral distance for various values of T as shown in Fig. 8(a), Fig. 8(b) and Fig. 8(c) respectively. The maximum and minimum values of the vertical displacement are given in Table 1. Vertical displacement assumes the maximum value in case of Maxwell model and minimum in case of Kelvin model for a particular value of time.

Fig. 9. shows the variation of the dimensionless stress τ_x for the three models namely Kelvin, Maxwell and SLS with the epicentral distance for various values of T as shown in Fig. 9(a), Fig. 9(b), and Fig. 9(c), respectively. We note that in case of Kelvin(Fig.9(a)) stress is same for various values of time except at T=0. In all the three cases, stress decreases first and then increases with the epicentral distance.

CONCLUSIONS

The explicit expressions for the displacements and stresses in an elastic and viscoelastic half-space due to a center of rotation source have been obtained by using Galerkin vector approach. Numerical results are shown graphically for displacements and stresses for different values of Poisson's ratio and three viscoelastic models namely Kelvin, Maxwell and SLS.

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REFERENCES

1. Love, AEH. A Treatise on the Mathematical Theory of Elasticity, Dover, New York, 1927.
2. Mindlin, RD. Force at A Point in The Interior of a Semi-Infinite Solid. J App Phys 1936; 195–202.
3. Mindlin, RD and Cheng, DH. Nuclei of Strain in The Semi-Infinite Solid. J App Phys 1950; 21: 926-930.
4. Okada, Y. Internal Deformation Due to Shear and Tensile Faults in a Half-Space. Bull Seism Soc Am 1992; 82: 1018–1040.
5. Singh, SJ and Rosenman, M. Quasi-static deformation of a viscoelastic half-space by a displacement dislocation. J Phys Earth Planet Int 1974; 8: 87–101.
6. Rundle, JB. Viscoelastic Crustal Deformation by Finite Quasi-Static Sources. J Geophys Res 1978; 83: 5937–5945.
7. Iwasaki, T and Matsu'ura, M. Quasi-Static Strains and Tilts Due to Faulting in a Layered Half-Space with an Intervenient Viscoelastic Layer. J Phys Earth 1981; 29: 499–518.
8. Iwasaki, T. Quasi-Static Deformation Due to a Dislocation Source in a Maxwellian Viscoelastic Earth Model. J Phys Earth 1985; 33: 21–43.
9. Bonafede, M, Dragoni, M and Quareni, F. Displacement and Stress Fields Produced by A Center of Dilatation and by A Pressure Source in A Viscoelastic Half-Space: Application to The Study of Ground Deformation and Seismic Activity at Campi Flegeri, Italy. Geophys J R AstrSoc 1986; 87: 455-485.
10. Singh, K and Singh, SJ.Static and Quasi-Static Deformation of a Uniform Half Space by Buried Sources. Geophy Res Bull 1989;27: 1-30.
11. Singh, K and Singh, SJ. Simple Procedure for Obtaining the Quasi-Static Displacement, Strains, And Stresses in a Viscoelastic Half-Space. Bull Seism Soc Am 1990;80: 488-492.
12. Singh, SJ, Kumari, G. and Singh, K. Static Deformation of Two Welded Elastic Half Spaces Caused by A Finite Rectangular Fault. Phys Earth Planet In 1993; 79: 313-333.
13. Singh, SJ, Kumari, G and Singh, K.Modelling of Crustal Deformation in Volcanic Areas. Nat Acad Sci Letters 1998; 21:165-176.

14. Cochard, A, Igel, H, Schuberth, B, et al. Rotational Motions in Seismology: Theory, Observations, Simulation, Earthquake Source Asymmetry, Sturctural Media and Rotation Effects, R. Teisseyre, M. Takeo, and E. Majewski(Editors), Springer, Berlin 2006; 391-412.
15. Cowsik, R, Nussinov, TM, Wagoner, K, et al. Performance Characteristics of a Rotational Seismometer for Near-Field and Engineering Applications. Bull Seism Soc Am 2009; 99: 1181-1189.
16. Trifunac, MD and Todorovska, MI. A note on the useable dynamic range of accelerographs recording translation. Soil Dyn and Earth Eng2001;21: 275-286.
17. Lee, WHK, Celebi, M, Todorovska, ML, et al. Introduction to the Special Issue on Rotational Seismology and Engineering Applications, Bull Seism Soc Am2009; 99:945-957.

Table 1: Below table containsthe maximum and minimum values of the vertical displacement.

Time(T)	Max.(Min.) value of vertical displacement (W)		
	Kelvin	Maxwell	SLS
0	0(0)	0.3(0)	0.35(0)
1	0.2(-0.21)	0.65(0)	0.49(0)
2	0.3(-0.3)	1(0)	0.55(0)
3	0.31(-0.31)	1.4(0)	0.62(0)

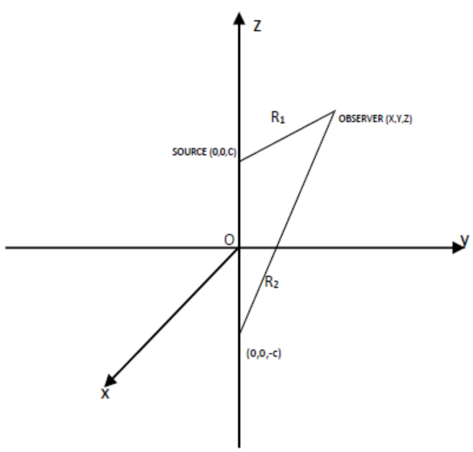

Figure 1: Geometry of a point source in a uniform half-space.

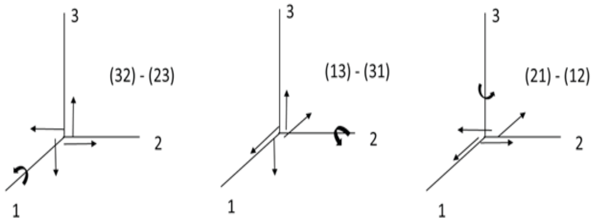

Figure 2: Three fundamental torques, $\Gamma^2 - \Gamma^3, \Gamma^3 - \Gamma^3, \Gamma^2 - \Gamma^2$ are center of rotations about the x_1, x_2 and x_3 axes respectively.

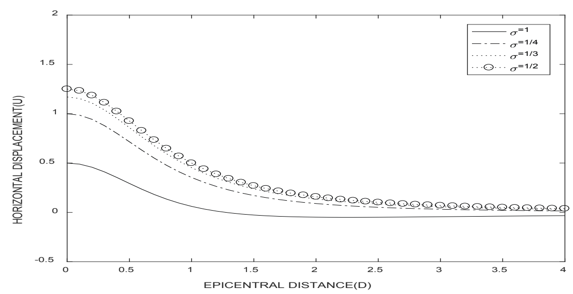

Figure 3: Variation of the dimensionless horizontal displacement (U) with the dimensionless epicentral distance (D) for various values of σ.

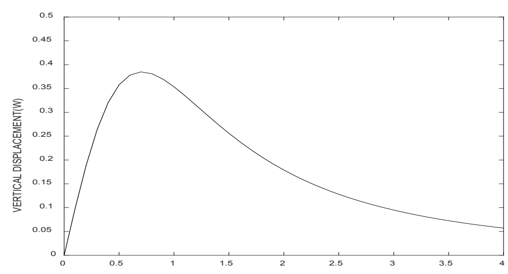

Figure 4: Variation of the dimensionless vertical displacement (W) with the dimensionless epicentral distance (D).

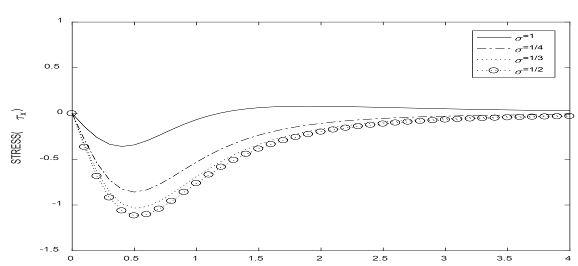

Figure 5: Variation of the dimensionless stress τ_x with the dimensionless epicentral distance for various values of σ.

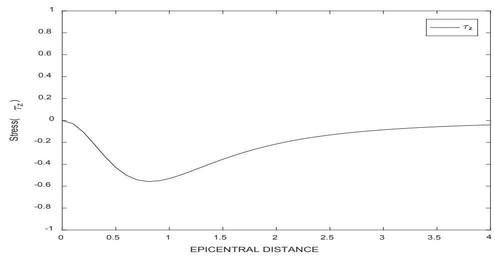

Figure 6: Variation of the dimensionless stress τ_z with the dimensionless epicentral distance.

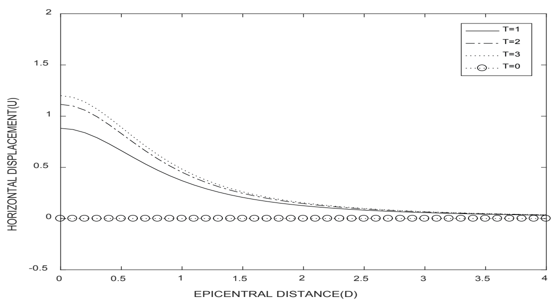

Figure 7(a): Variation of the dimensionless horizontal displacement with the dimensionless epicentral distance for the various values of T for kelvin model.

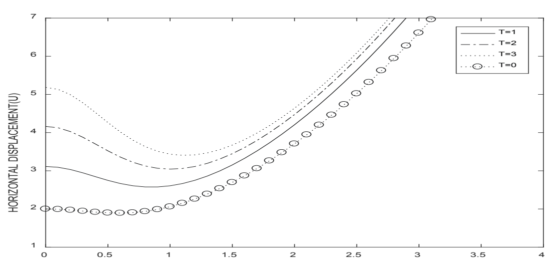

Figure 7(b): Variation of the dimensionless horizontal displacement with the dimensionless epicentral distance for various values of T for Maxwell model.

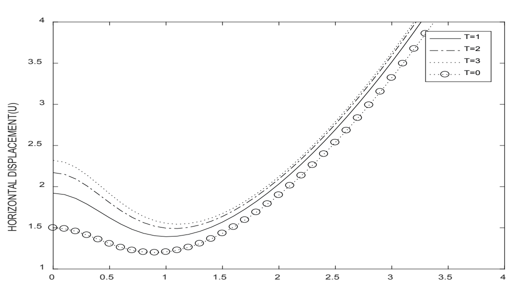

Figure 7(c): Variation of the dimensionless horizontal displacement with the dimensionless epicentral distance for various values of T for SLS model.

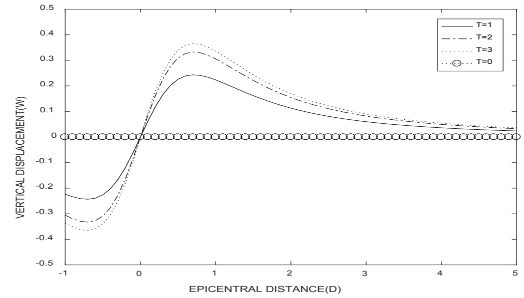

Figure 8(a): Variation of the dimensionless vertical displacement with the dimensionless epicentral distance for various values of T for kelvin model.

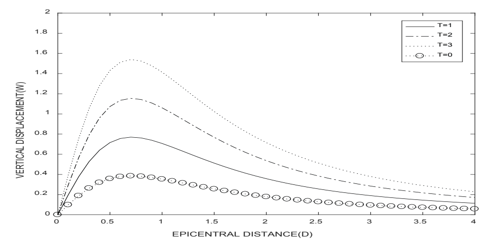

Figure 8(b): Variation of the dimensionless vertical displacement with the dimensionless epicentral distance for various values of T for Maxwell model.

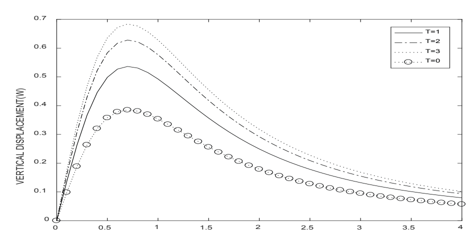

Figure 8(c): Variation of the dimensionless vertical displacement with the dimensionless epicentral distance for various values of T for SLS model.

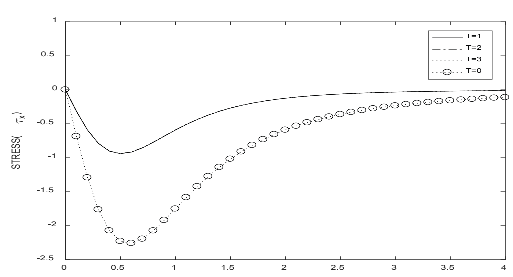

Figure 9(a): Variation of the dimensionless stress τ_x with the dimensionless epicentral distance for various values of T for kelvin model.

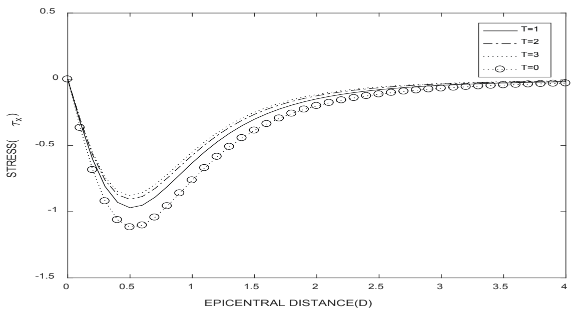

Figure 9(b): Variation of the dimensionless stress τ_x with the dimensionless epicentral distance for various values of T for Maxwell model.

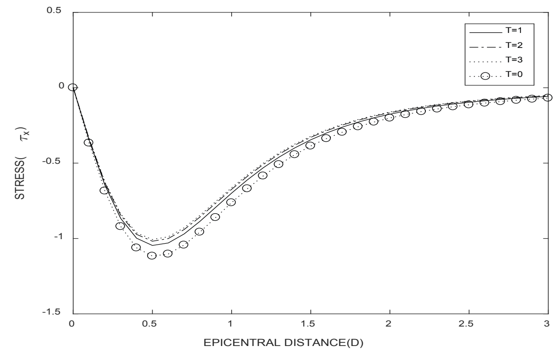

Figure 9(c): Variation of the dimensionless stress τ_x with the dimensionless epicentral distance for various values of T for SLS model.