IJCRR
Vol 04 issue 22
Section: General Sciences
Category: Research
Received on: 20/10/12
Revised on: 28/10/12
Accepted on: 05/11/12

A CHARACTERIZATION OF THERMOSOLUTAL INSTABILITY IN RIVLIN-ERICKSEN ROTATING FLUID IN A POROUS MEDIUM

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ABSTRACT

Thermosolutal instability of Veronis (1965) type in Rivlin-Ericksen viscoelastic fluid in the presence of uniform vertical rotation in a porous medium is considered. The paper established the condition for characterizing the oscillatory motions which may be neutral or unstable, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension and finite vertical depth in a porous medium, are necessarily non-oscillatory, in the regime

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A P l^2 \leq 1,$$

where R_s is the Thermosolutal Rayliegh number, T_A is the Taylor number, p_2 is the magnetic Prandtl number, p_3 is the thermosolutal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the viscoelasticity parameter. The result is important since it hold for all wave numbers and for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid. A similar characterization theorem is also proved for Stern (1960) type of configuration.

Keywords: Thermal convection; Rivlin-Ericksen Fluid; Rotation; Rayleigh number; Taylor number.

INTRODUCTION

The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics, and has been investigated by several authors (e.g., Bénard [4], Rayleigh [13], Jeffreys [8]) under different conditions. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [6] in his celebrated monograph. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by Veronis [19]. The physics is quite similar in the stellar case, in that helium acts like in raising the density and in diffusing more slowly than heat. The condition under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore it is desirable to consider a fluid acted upon by a solute gradient with free or rigid boundaries. The problem is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering. Bhatia and Steiner [6] have considered the effect of uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. Sharma [16] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted upon by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's [11] constitutive relations. Two such classes of fluids are Rivlin-Ericksen's and Walter's (model B') fluids. Rivlin-Ericksen [14] has proposed a theoretical model for such one class of elastico-viscous fluids. Sharma and kumar [17] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Kumar et al. [9] considered effect of rotation and magnetic field on Rivlin-Ericksen elastico-viscous fluid and found that rotation has stabilizing effect; where as magnetic field has both stabilizing and destabilizing effects. A layer of such fluid heated from below or under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable.

In all above studies, the medium has been considered to be non-porous with free boundaries only, in general. In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. When a fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equation of Rivlin-Ericksen fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)q\right]$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of great importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of the comets, meteorites and interplanetary dust strongly suggest the importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. Thermal convection in porous medium is also of interest in geophysical system, electrochemistry and metallurgy. A comprehensive review of the literature concerning thermal convection in a fluid-saturated porous medium may be found in the book by Nield and Bejan [10].

Pellow and Southwell [12] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [2] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee $[1]$ established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al $[7]$. However no such result existed for non-Newtonian fluid configurations in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal $[3]$ have characterized the oscillatory motions in Rivlin-Ericksen fluid in the presence of magnetic field.

Keeping in mind the importance of non-Newtonian fluids, as stated above, this article attempts to study Rivlin-Ericksen viscoelastic of Veronis and Stern type configuration in the presence of uniform vertical rotation in a porous medium, and it has been established that the onset of instability in a Rivlin-Ericksen viscoelastic fluid heated from below in a porous medium Veronis type configuration, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the viscoelasticity parameter F satisfy the inequality $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2 \leq 1$, for all wave numbers and for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid. A similar characterization theorem is also proved for Stern type of configuration, for all wave numbers and for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid.

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we Consider an infinite, horizontal, incompressible Rivlin-Ericksen viscoelastic fluid layer, of thickness d, heated from below so that, the temperature, density and solute concentrations at the bottom surface z = 0 are T_0, ρ_0 and C_0 at the upper surface z = d are T_d, ρ_d and C_d respectively, and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ and a uniform solute gradient $\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ is maintained. The gravity field $\vec{g}(0,0,-g)$ and uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ pervade on the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let p, ρ, T, C, α, α', g and $\vec{q}(u,v,w)$ denote respectively the fluid pressure, fluid density temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and filter velocity of the fluid. Then the momentum balance, mass balance, and energy balance equation governing the flow of Rivlin-Ericksen fluid in the presence of uniform vertical vertical rotation (Rivlin and Ericksen $[14]$; Chandrasekhar $[6]$ and Sharma et al $[18]$) are given by

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}.\nabla\right)\vec{q}\right]=-\left(\frac{1}{\rho_0}\right)\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \qquad (1)$$

$$\nabla.\vec{q}=0, \qquad (2)$$

$$E\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \qquad (3)$$

And

$$E'\frac{\partial C}{\partial t}+(\vec{q}.\nabla)C=\kappa'\nabla^2 C \tag{4}$$

Where $\frac{d}{dt}=\frac{\partial}{\partial t}+\varepsilon^{-1}\vec{q}.\nabla$, stands for the convective derivatives. Here $E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_i}\right)$, is a constant and E' is a constant analogous to E but corresponding to solute rather than heat, while ρ_s, c_s and ρ_0, c_i, stands for the density and heat capacity of the solid (porous matrix) material and the fluid, respectively, ε is the medium porosity and $\vec{r}(x,y,z)$.

The equation of state is

$$\rho=\rho_0\left[1-\alpha(T-T_0)+\alpha'(C-C_0)\right], \tag{5}$$

Where the suffix zero refer to the values at the reference level z = 0. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The kinematic viscosity ν, kinematic viscoelasticity ν', thermal diffusivity κ, the solute diffusivity κ' and the coefficient of thermal expansion α are all assumed to be constants.

The steady state solution is

$$\vec{q}=(0,0,0),\ \rho=\rho_0(1+\alpha\beta z-\alpha'\beta' z),$$

$$T=-\beta z+T_0,\ C=-\beta' z+C_0, \tag{6}$$

Here we use the linearized stability theory and the normal mode analysis method. Consider a small perturbations on the steady state solution, and let $\delta\rho$, δp, θ, γ and $\vec{q}(u,v,w)$ denote respectively the perturbations in density ρ, pressure p, temperature T, solute concentration C and velocity $\vec{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbation θ and γ in temperature and concentration, is given by

$$\delta\rho=-\rho_0(\alpha\theta-\alpha'\gamma) \tag{7}$$

Then the linearized perturbation equations of the Rinlin-Ericksen fluid reduces to

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_0}(\nabla\delta p)-\vec{g}(\alpha\theta-\alpha'\gamma)-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{8}$$

$$\nabla.\vec{q}=0, \tag{9}$$

$$E\frac{\partial\theta}{\partial t}=\beta w+\kappa\nabla^2\theta, \tag{10}$$

And

$$E'\frac{\partial\gamma}{\partial t}=\beta' w+\kappa'\nabla^2\gamma, \tag{11}$$

NORMAL MODE ANALYSIS

Analyzing the disturbances into two-dimensional waves, and considering disturbances characterized by a particular wave number, we assume that the Perturbation quantities are of the form

$$[w,\theta,\gamma,\varsigma]=[W(z),\Theta(z),\Gamma(z),Z(z)]\exp\left(ik_x x+ik_y y+nt\right), \tag{12}$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k=\left(k_x{}^2+k_y{}^2\right)^{\frac{1}{2}}$, is the resultant wave number, n is the growth rate which is, in general, a complex constant and $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ denote the z-component of vorticity; $W(z), \Theta(z), \Gamma(z)$ and $Z(z)$ are the functions of z only.

Using (12), equations (8)-(11), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]\left(D^2 - a^2\right)W = -Ra^2\Theta + R_s a^2\Gamma - T_A DZ\,, \tag{13}$$

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]Z = DW\,, \tag{14}$$

$$\left(D^2 - a^2 - Ep_1\sigma\right)\Theta = -W\,, \tag{15}$$

And

$$\left(D^2 - a^2 - E' p_3\sigma\right)\Gamma = -W\,, \tag{16}$$

Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$ is the thermal Prandtl number; $p_3 = \frac{\nu}{\kappa'}$ is the thermosolutal Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\nu'}{d^2}$ is the dimensionless viscoelasticity parameter of the Rivlin-Ericksen fluid; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$ is the thermal Rayleigh number; $R_s = \frac{g\alpha'\beta' d^4}{\kappa'\nu'}$ is the thermosolutal Rayleigh number; and $T_A = \frac{4\Omega^2 d^4}{\nu^2\varepsilon^2}$ is the Taylor number. Also we have Substituted $W = W_\oplus, \Theta = \frac{\beta d^2}{\kappa}\Theta_\oplus$, $\Gamma = \frac{\beta' d^2}{\kappa}\Gamma_\oplus$, $Z = \frac{2\Omega d}{\nu\varepsilon}Z_\oplus$ and $D_\oplus = dD$ and dropped $(\oplus)$ for convenience.

We now consider the cases where the boundaries are rigid-rigid or rigid-free or free-rigid or free-free at z = 0 and z = 1 respectively, as the case may be, and are maintained at constant temperature and solute concentration. Then the perturbations in the temperature and solute concentration are zero at the boundaries. The appropriate boundary conditions with respect to which equations (13)--(16), must possess a solution are

$W = 0 = \Theta = \Gamma$, on both the horizontal boundaries,

$DW = 0 = Z$, on a rigid boundary,

$D^2W = 0 = DZ$, on a dynamically free boundary, (17)

Equations (13)--(16), along with boundary conditions (17), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, Γ and Θ satisfy $W(0) = 0 = W(1)$, $\Gamma(0) = 0 = \Gamma(1)$ and $\Theta(0) = 0 = \Theta(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality Schultz [15]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz \,, \int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz \text{ and}$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz \,, \tag{18}$$

MATHEMATICAL ANALYSIS

We prove the following Lemma's:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{\pi^4 a^2} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (16) by Γ^* (the complex conjugate of Γ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition (17) on Γ namely $\Gamma(0) = 0 = \Gamma(1)$, it follows that

$$\int_0^1 \left\{ |D\Gamma|^2 + a^2 |\Gamma|^2 \right\} dz + E' \sigma_r p_3 \int_0^1 |\Gamma|^2 dz = \text{Real part of} \left\{ \int_0^1 \Gamma^* W dz \right\},$$

$$\leq \left| \int_0^1 \Gamma^* W dz \right| \leq \int_0^1 |\Gamma^* W| dz \leq \int_0^1 |\Gamma^*| |W| dz \,,$$

$$\leq \int_0^1 |\Gamma| |W| dz \leq \left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{19}$$

(Utilizing Cauchy-Schwartz-inequality),

This gives that

$$a^2 \int_0^1 |\Gamma|^2 dz \leq \left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{ \int_0^1 |\Gamma|^2 dz \right\}^{\frac{1}{2}} \leq \frac{1}{a^2} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{20}$$

Since $\sigma_r \geq 0$ and $p_3 \rangle 0$, hence inequality (19) on utilizing (18) and (20), gives

$$\int_0^1 |\Gamma|^2 dz \leq \frac{1}{a^2 \pi^4} \int_0^1 |DW|^2 dz \,, \tag{21}$$

This completes the proof of lemma**.**

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2 \pi^4} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (15) by Θ^* (the complex conjugate of Θ), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition (17) on Θ namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\int_0^1 \left\{ |D\Theta|^2 + a^2 |\Theta|^2 \right\} dz + E \sigma_r p_1 \int_0^1 |\Theta|^2 dz = \text{Real part of} \left\{ \int_0^1 \Theta^* W dz \right\},$$

$$\leq \left| \int_0^1 \Theta^* W dz \right| \leq \int_0^1 |\Theta^* W| dz \leq \int_0^1 |\Theta^*| |W| dz \,,$$

$$\leq \int_0^1 |\Theta| |W| dz \leq \left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{22}$$

(Utilizing Cauchy-Schwartz-inequality),

This gives that

$$a^2 \int_0^1 |\Theta|^2 dz \leq \left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}},$$

And thus, we get

$$\left\{ \int_0^1 |\Theta|^2 dz \right\}^{\frac{1}{2}} \leq \frac{1}{a^2} \left\{ \int_0^1 |W|^2 dz \right\}^{\frac{1}{2}}, \tag{23}$$

Since $\sigma_r \geq 0$ and $p_1 \rangle 0$, hence inequality (22) on utilizing (23) and (18), gives

$$\int_0^1 |\Theta|^2 dz \leq \frac{1}{a^2 \pi^4} \int_0^1 |DW|^2 dz \,, \tag{24}$$

This completes the proof of lemma 2.

Lemma 3: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \leq P_l^2 \int_0^1 |DW|^2 dz \cdot$$

Proof: Multiplying equation (14) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and utilizing appropriate boundary conditions (17), it follows that

$$\left[\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}(1+\sigma_r F)\right]\int_0^1 |Z|^2 dz$$

=Real part of $\left\{\int_0^1 DW^* Z dz\right\} \leq \left|\int_0^1 DW^* Z dz\right|$,

$$\leq \int_0^1 |DW^* Z| dz \leq \int_0^1 |DW^*||Z| dz,$$

$$= \int_0^1 |DW||Z| dz \leq \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \quad (25)$$

(Utilizing Cauchy-Schwartz-inequality),

This gives that

$$\frac{1}{P_l}\int_0^1 |Z|^2 dz \leq \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}},$$

This implies that

$$\left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \leq P_l \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}}, \quad (26)$$

Since $\sigma_r \geq 0$ and $p_2 \rangle 0$, hence inequality (25), on utilizing inequality (26) give

$$\int_0^1 |Z|^2 dz \leq P_l^2 \int_0^1 |DW|^2 dz, \quad (27)$$

This completes the proof of lemma**.**

We prove the following theorem:

Theorem 1: If $R \rangle 0$, $R_s \rangle 0$, $F \rangle 0$, $T_A \rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $p_3 \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, Γ, Z) of equations (13) – (16), together with boundary conditions (17) is that

$$\left(\frac{\varepsilon P_l}{P_l + \varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right) + T_A P_l^2 \rangle 1 \cdot$$

Proof: Multiplying equation (13) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]\int_0^1 W^*\left(D^2 - a^2\right)W dz = -Ra^2 \int_0^1 W^* \Theta dz$$

$$+ R_s a^2 \int_0^1 W^* \Gamma dz - T_A \int_0^1 W^* DZ dz, \quad (28)$$

Taking complex conjugate on both sides of equation (15), we get

$$\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* = -W^*, \quad (29)$$

Therefore, using (29), we get

$$\int_0^1 W^* \Theta dz = -\int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz, \quad (30)$$

Taking complex conjugate on both sides of equation (16), we get

$$\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* = -W^*, \quad (31)$$

Therefore, using (31), we get

$$\int_0^1 W^* \Gamma dz = -\int_0^1 \Gamma\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma^* dz, \quad (32)$$

Also taking complex conjugate on both sides of equation (14), we get

$$\left[\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}(1+\sigma^* F)\right] Z^* = DW^*, \quad (33)$$

Therefore, using (33), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = -\left[\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}(1+\sigma^* F)\right]\int_0^1 Z^* Z dz, \quad (34)$$

Substituting (30), (32) and (34), in the right hand side of equation (28), we get

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+\sigma F)\right]\int_0^1 W^*\left(D^2 - a^2\right)W dz = Ra^2 \int_0^1 \Theta\left(D^2 - a^2 - Ep_1\sigma^*\right)\Theta^* dz - R_s a^2 \int_0^1 \Gamma^*\left(D^2 - a^2 - E' p_3\sigma^*\right)\Gamma dz$$

$$+T_A\left[\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}(1+\sigma^* F)\right]\int_0^1 Z^* Z dz\,, \qquad (35)$$

Integrating the terms on both sides of equation (35) for an appropriate number of times and making use of the appropriate boundary conditions (17), we get

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1+\sigma F)\right]\int_0^1\left(|DW|^2+a^2|W|^2\right)dz = Ra^2\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2+Ep_1\sigma^*|\Theta|^2\right)dz$$

$$-R_s a^2\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2+E' p_3\sigma^*|\Gamma|^2\right)dz - T_A\left[\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}(1+\sigma^* F)\right]\int_0^1 |Z|^2 dz\,, \qquad (36)$$

Now equating imaginary parts on both sides of equation (42), and cancelling $\sigma_i (\neq 0)$, we get

$$\left[\frac{1}{\varepsilon}+\frac{F}{P_l}\right]\int_0^1\left(|DW|^2+a^2|W|^2\right)dz = \left[-Ra^2Ep_1\int_0^1|\Theta|^2 dz + R_s a^2 E' p_3\int_0^1|\Gamma|^2 dz + T_A\left\{\frac{1}{\varepsilon}+\frac{F}{P_l}\right\}\int_0^1|Z|^2 dz\right], \qquad (37)$$

Now R $\rangle$ $0, \varepsilon\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (21) and (27), the equation (37) gives,

$$\left[\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)-T_A P_l^2\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)-\frac{R_s E' p_3}{\pi^4}\right]\int_0^1 |DW|^2 dz + I_1\langle 0\,, \qquad (38)$$

Where

$$I_1=\left(\frac{1}{\varepsilon}+\frac{F}{P_l}\right)a^2\int_0^1|W|^2 dz + Ra^2Ep_1\int_0^1|\Theta|^2 dz\,,$$

Is positive definite, and therefore, we must have

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2\rangle 1. \qquad (39)$$

Hence, if

$$\sigma_r \geq 0 \text{ and } \sigma_i \neq 0, \text{ then } \left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2\rangle 1. \qquad (40)$$

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 1, can be put in the form as follow:-

Corollary 1: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Veronis type in the presence of uniform rotation in a porous medium heated from below is that, $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2 \leq 1$, where R_s is the Thermosolutal Rayliegh number, T_A is the Taylor number, p_2 is the magnetic Prandtl number, p_3 is the thermosolutal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the viscoelasticity parameter, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Veronis type in the presence of uniform vertical rotation in a porous medium heated from below, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the viscoelasticity parameter F, satisfy the inequality $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2 \leq 1$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 2: If $(W,\Theta,\Gamma,Z,\sigma)$, $\sigma=\sigma_r+i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (15) – (19), with $R \rangle\ 0$ and,

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2 \leq 1 ,$$

Then $\sigma_i=0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r=0 \Rightarrow \sigma_i=0$ if $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2 \leq 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Veronis type, we can state the above theorem as follow:-

Corollary 3: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Veronis type in the presence of uniform vertical rotation in a porous medium heated from below is that the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the viscoelasticity parameter F must satisfy the inequality $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2 \rangle 1$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid.

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in Rivlin Ericksen viscoelastic fluid heated from below, i. e. when $T_A=0=R_s$ Sharma $[16]$.

(ii). Rotatory-thermal convection in Rivlin Ericksen viscoelastic fluid heated from below (R_s=0), if

$$T_A \leq \frac{1}{P_l^2} .$$

(iii) Thermosolutal convection of Veronis (1965) type in Rivlin Ericksen viscoelastic fluid heated from below (T_A = 0), if

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right) \leq 1 .$$

A similar theorem can be proved for thermosolutal convection in rivlin-Ericksen Viscoelastic fluid configuration of Stern type in a porous medium as follow:

Theorem 2: If $R\langle\ 0$, $R_s \langle 0$, $F\rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $p_3 \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution $(W,\Theta,\Gamma,Z,\sigma)$ of equations (13) – (16), together with boundary conditions (17) is that

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R| E p_1}{\pi^4}\right)+T_A {P_l}^2 \rangle 1 .$$

Proof: Replacing R and R_s by $-|R|$ and $-|R_s|$, respectively in equations (17) – (20) and proceeding exactly as in Theorem 1 and utilizing the inequality (28), we get the desired result.

Presented otherwise from the point of view of existence of instability as stationary convection, the above Theorem 2, can be put in the form as follow:-

Corollary 4: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Stern type in the presence of uniform vertical rotation in a porous medium is that, $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R|Ep_1}{\pi^4}\right)+T_A P_l{}^2 \le 1$, where R is the Thermal Rayliegh number, the Taylor number T_A, p_2 is the magnetic Prandtl number, p_1 is the thermal Prandtl number, P_l is the medium permeability, ε is the porosity and F is the viscoelasticity parameter, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

or

The onset of instability in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Stern type in the presence of uniform vertical rotation in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermal Rayliegh number R, the Taylor number T_A, the magnetic Prandtl number p_2, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the viscoelasticity parameter F, satisfy the inequality $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R|Ep_1}{\pi^4}\right)+T_A P_l{}^2 \le 1$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Corollary 5: If $(W,\Theta,\Gamma,Z,\sigma)$, $\sigma=\sigma_r+i\sigma_i$, $\sigma_r \ge 0$ is a solution of equations (17) – (20), with $R \rangle 0$ and,

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R|Ep_1}{\pi^4}\right)+T_A P_l{}^2 \le 1 ,$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R|Ep_1}{\pi^4}\right)+T_A P_l{}^2 \le 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration of Stern's type, we can state the above theorem as follow:-

Corollary 6: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Stern type in the presence of uniform vertical rotation in a porous medium is that the Thermal Rayliegh number R, the Taylor number T_A, the magnetic Prandtl number p_2, the thermal Prandtl number p_1, the medium permeability P_l, the porosity ε and the viscoelasticity parameter F must satisfy the inequality $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R|Ep_1}{\pi^4}\right)+T_A P_l{}^2 \rangle 1$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid.

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in Rivlin Ericksen viscoelastic fluid i. e. when T_A = 0 =R.

(ii). Royatory-thermal convection Rivlin Ericksen viscoelastic fluid (R=0), if

$$T_A \le \frac{1}{P_l{}^2}.$$

(iii). Thermosolutal convection of Stren (1960) type in Rivlin Ericksen viscoelastic fluid (T_A = 0), if

$$\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{|R|Ep_1}{\pi^4}\right)\le 1 \; .$$

CONCLUSIONS

Theorem 1 mathematically established that the onset of instability in a thermosolutal Rivlin-Ericksen viscoelastic fluid configuration of Veronis (1965) type in the presence of uniform vertical rotation in a porous medium, cannot manifest itself as oscillatory motions of growing amplitude if the Thermosolutal Rayliegh number R_s, the Taylor number T_A, the magnetic Prandtl number p_2, the thermosolutal Prandtl number p_3, the medium permeability P_l, the porosity ε and the viscoelasticity parameter F satisfy the inequality $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2\le 1$, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid

The essential content of the theorem 1, from the point of view of linear stability theory is that for the thermosolutal configuration of Veronis (1965) type of Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension in the presence of uniform vertical rotation in a porous medium, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\left(\frac{\varepsilon P_l}{P_l+\varepsilon F}\right)\left(\frac{R_s E' p_3}{\pi^4}\right)+T_A {P_l}^2\le 1$, and in particular PES is valid.

The similar conclusions can be drawn for the thermosolutal configuration of Stern (1960) type of Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension in the presence of uniform vertical rotation in a porous medium, for any arbitrary combination of free and rigid boundaries at the top and bottom of the fluid from Theorem 2.

ACKNOWLEDGEMENT

Author acknowledges the immense help received from the scholars whose articles are cited and included in references of this manuscript. The authors are also grateful to authors / editors /publishers of all those articles, journals and books from where the literature for this article has been reviewed and discussed. The author is highly thankful to the referees for their very constructive, valuable suggestions and useful technical comments, which led to a significant improvement of the paper.

REFERENCES

1. Banerjee, M. B., and Banerjee, B., A characterization of non-oscillatory motions in magnetohydronamics, Ind. J. Pure & Appl Maths., 1984, 15(4): 377-382
2. Banerjee, M.B., Katoch, D.C., Dube G.S. and Banerjee, K., Bounds for growth rate of perturbation in thermohaline convection. Proc. R. Soc. A, 1981,378, 301-04
3. Banyal, A.S, A characterization of Rivlin-Ericksen viscoelastic fluid in the presence of magnetic field, Int. J. of Mathematical Archives, Vol. 3(7), 2012, pp. 2543-2550.
4. Bénard, H., Les tourbillions cellulaires dans une nappe liquid, Revue Genérale des Sciences Pures et Appliquees 11 (1900), 1261-1271, 1309-1328.
5. Bhatia, P.K. and Steiner, J.M., Convective instability in a rotating viscoelastic fluid layer, Zeitschrift fur Angewandte Mathematik and Mechanik 52 (1972), 321-327.
6. Chandrasekhar, S. Hydrodynamic and Hydromagnetic Stability, 1981, Dover Publication, New York.
7. Gupta, J.R., Sood, S.K., and Bhardwaj, U.D., On the characterization of nonoscillatory motions in rotatory hydromagnetic thermohaline convection, Indian J. pure appl.Math. 1986,17(1), pp 100-107.

8. Jeffreys, H., The stability of a fluid layer heated from below, Philosophical Magazine 2 (1926), 833-844.
9. Kumar, P., Mohan, H. and Lal, R., Effect of magnetic field on thermal instability of a rotating Rivlin-Ericksen viscoelastic fluid, Int. J. of Maths. Math. Scs., Vol-2006 article ID 28042, pp. 1-10.
10. Nield D. A. and Bejan, A., Convection in porous medium, springer, 1992.
11. Oldroyd, J.G., Non-Newtonian effects in steady motion of some idealized elastic-viscous liquids, Proceedings of the Royal Society of London A245 (1958), 278-297.
12. Pellow, A., and Southwell, R.V., On the maintained convective motion in a fluid heated from below. Proc. Roy. Soc. London A, 1940, 176, 312-43
13. Rayleigh, L., On convective currents in a horizontal layer of fluid when the higher temperature is on the underside, Philosophical Magazine 32 (1916), 529-546.
14. Rivlin, R.S. and Ericksen, J. L., Stress deformation relations for isotropic materials, J. Rat. Mech. Anal. 4 (1955), 323.
15. Schultz, M.H. (1973)., Spline Analysis, Prentice Hall, Englewood Cliffs, New Jersy.
16. Sharma, R.C., Effect of rotation on thermal instability of a viscoelastic fluid, Acta Physica Hungarica 40 (1976), 11-17.