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A Heuristic Approach to Minimize Utilization Time in N × 2 Specially Structured Flow Shop Scheduling Problem Including Setup Time, Transportation Time and Jobs in a String of Disjoint Job Blocks

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ABSTRACT

The present paper is an attempt to obtain a sequence of jobs through heuristic method to optimize the utilization time of machines for specially structured n-job and 2-machine flow shop scheduling problem. Also, the jobs are to be processed in a string of disjoint job blocks having sequence independent setup times separated from processing times each associated with their respective probabilities including transportation time. In flow shop scheduling minimization of total elapsed time may not always result in minimization of utilization time of machines. To minimize the utilization time an algorithm is proposed and a numerical problem is solved to authenticate the algorithm.

Key Words: Elapsed time, Expected processing time, Expected setup time, Jobs in a String

INTRODUCTION

Scheduling models deals with determination of an optimal sequence to provide service to customers or to perform a set of jobs in order to minimize the total elapsed time or some other suitable performance measure. In today's manufacturing and distribution systems, scheduling have significant role to meet customer requirements as quickly as possible while maximizing the profits. In flow shop scheduling problem n-jobs are processed on m-machines and the processing order i.e. the order in which various machines are required for completing the job is given. The common objectives in flow shop scheduling problems are to minimize some performance measures such as make span, mean flow time, mean tardiness, mean setup time, number of tardy jobs and mean number of setups. Johnson[(1)] developed an algorithm for two stage production schedule for minimizing the make span. Palmer DS[(2)] developed a heuristic algorithm for sequencing jobs to minimize the total elapsed time. Gupta JND[(3)] studied specially structured flow shop scheduling problem to obtain an optimal sequence of jobs. Gupta D, Sharma S & Bala S[(4)] investigated specially structured two stage flow shop scheduling problem under rental situation.

Corwin BD et al[(5)] studied two machine flow shop scheduling problems with sequence dependent setup time. Setup time includes the time to prepare the machines, obtaining, adjusting and returning tools for an operation, cleaning up the machines, setting the necessary jigs and fixtures and inspecting and positioning the process material. Setup time has an important part as reduction in setup time leads to increase in output, profitability and customer satisfaction in an organization. The setup times in scheduling problems can be classified into two categories viz. sequence-independent setup times and sequence-dependent setup times. Sequence-independent setup time depends solely on the current job to be

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processed regardless of previously processed job. Sequence dependent setup time depend both on the current job and previously processed job.

In most manufacturing and distribution systems, semi finished jobs are moved from one processing facility to another for further processing and finished jobs are delivered to customers or store houses by suitable modes. In job sequencing the time required in moving a job from one machine to another machine during the processing of jobs is known as transportation time. Maggu and Dass[6] considered a two machine flow shop problem including transportation time of jobs from first machine to the second machine. Langston[7] gave heuristic solution to minimize make span for a k-station flow shop problem where each station has a number of machines that can be used to process jobs and there is only one transporter with a capacity to transport one job with transportation times dependent on the physical locations of the starting and destination machines. Chung et al[8] studied machine scheduling problems with explicit transportation considerations. They considered scheduling models for transporting a semi-finished job from one machine to another for further processing and for delivering finished jobs to the customers or warehouses. Gupta D, Sharma S and Bala S[9] applied heuristic algorithm to minimize the utilization time and rental cost of machines for two stage specially structured flow shop scheduling problem involving transportation time.

Maggu and Das[10] established the basic concept of equivalent job for job block in scheduling theory. The string of disjoint job blocks consist of two disjoint job blocks such that in one job block the order of jobs is fixed and in second job block the order of jobs is arbitrary. Anup and Maggu[11] gave an optimal schedule for n × 2 flow shop problem with job blocks of jobs in which first job in each job block being the same. Heydari[12] studied flow shop scheduling problem with processing of jobs in a string of disjoint job blocks. Singh TP, Kumar V and Gupta D[13] studied n × 2 flow-shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks. Gupta D, Sharma S and Gulati N[14] studied n×3 flow-shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks. Gupta et al[15] considered specially structured two stage flow shop scheduling problem having jobs in a string of disjoint job blocks.

In this paper we investigate n-job and two machines specially structured flow shop scheduling problem with sequence independent setup times separated from processing times each associated with probabilities including transportation time and jobs to be processed as string of disjoint job blocks. The aim of the study is to obtain a sequence of jobs that optimizes the utilization time of machines. An algorithm is proposed to solve the problem and is validated with the help of a numerical example.

PRACTICAL SITUATION

The service centres and industrial units must utilize their resources in an optimal manner to increase their profits and productivity. For optimal utilization of available resources there must be a proper scheduling system and this makes scheduling a highly important aspect of industrial establishments and service sectors. Specially structured two machine flow shop scheduling problem has been considered as there are many realistic situations where the processing times of jobs on the two machines are related in specific manner. In many practical situations such as chemical, food processing, pharmaceutical, metal processing, printing etc. setup time is required while shifting from one operation to another.

The idea of job block has practical importance to deal with ordering of jobs so as to ensure priority in service to the preferred customers and/or jobs and hence maximize profits. Scheduling models with jobs in a string of disjoint job blocks are necessary in cases where certain orderings of jobs are prescribed either by technological constraints or by externally imposed policy[10].

During the processing of jobs in many production and distribution units, semi-finished tasks are transferred from one machine to another through various modes such as automated guided vehicles and conveyors, and finished jobs are delivered to consumers or storehouses by vehicles such as trains or trucks. Machine scheduling models that take into account the job transportation time are indeed more realistic than those scheduling models that do not take into consideration these parameters.

NOTATIONS

The following notations have been used throughout the paper:

σ: Sequence of n- jobs obtained by applying Johnson's algorithm.

σ_k: Sequence of jobs obtained by applying the proposed algorithm, k = 1, 2, 3, ------.

M_j: Machine j, j= 1, 2.

a_{ij}: Processing time of i^{th} job on machine M_j.

s_{ij}: Set up time of i^{th} job on machine M_j.

p_{ij}: Probability associated to the processing time a_{ij}.

q_{ij}: Probability associated to the set up time s_{ij}.

A_{ij}: Expected processing time of i^{th} job on machine M_j.

S_{ij}: Expected set up time of i^{th} job on machine M_j.

$T_{i,1\to2}$: Transportation time of i^{th} job from first machine to second machine.

$t_{ij}(\sigma_k)$: Completion time of i^{th} job of sequence σ_k on machine M_j.

$T(\sigma_k)$: Total elapsed time for jobs 1, 2, --------, n for sequence σ_k.

$U_j(\sigma_k)$: Utilization time for which machine M_j is required for sequence σ_k.

$A_{ij}(\sigma_k)$: Expected processing time of i^{th} job on machine M_j for sequence σ_k.

α: Fix order job block.

β: Job block with arbitrary order.

β_k: Job block with jobs in an optimal order obtained by applying the proposed algorithm,

k = 1, 2, 3, ------.

S: String of job blocks α and β i.e. S = (α, β)

S': Optimal string of job blocks α and β_k.

ASSUMPTIONS

The assumptions for the proposed algorithm are stated below:

a) Jobs are independent to each other and are processed thorough two machines M_1 and M_2 in order M_1 M_2.
b) Pre-emption is not allowed. Once a job is started on a machine the process on that machine cannot be stopped unless the job is completed.
c) Processing times must satisfy the structural conditions $\min_i \{G_i\} \geq \max_i \{H_i\}$ or $\max_i \{G_i\} \leq \min_i \{H_i\}$.
d) Each job has two operations and each job is processed through each of the machine once and only once.
e) The independency of processing times of jobs on the schedule is maintained.
f) Only one machine of each type is available.
g) $\sum_{i=1}^{n}{}_{ij} = 1$, $\sum_{i=1}^{n}{}_{ij} = 1$, $0 \leq p_{ij}, q_{ij} \leq 1$
h) Jobs i_1, i_2, ---------------, i_h are to be processed as a job block (i_1, i_2, ----------------, i_h) showing priority of job i_1 over i_2 etc. in that order in case of a fixed order job block.

DEFINITION

Completion time of i^{th} job on machine M_j is given by,

$$t_{ij} = \max(t_{i-1,j} + S_{i-1,j},\ t_{i,j-1} + t_{i,1\to 2}) + A_{ij};\ j \geq 2,$$

where A_{ij} = Expected processing time of i^{th} job on machine M_j and S_{ij} = Expected set up time of i^{th} job on machine M_j.

THEOREM

If $A_{i1} \geq A_{j2}$ for each i and j, then K_1, K_2, ----------, K_n is a monotonically increasing sequence, where $K_n = \sum_{i=1}^{n} A - \sum_{i=1}^{n-1} A$.

Proof: Let $K_n = \sum_{i=1}^{n} A - \sum_{i=1}^{n-1} A$

Let $A_{i1} \geq A_{j2}$ for each i and j.

Thus, we have $\min_i A_{i1} \geq \max_j A$.

Here $K_1 = A_{11}$.

Now $K_2 = A_{11} + A_{21} - A_{12} = A_{11} + (A_{21} - A_{12}) \geq K_1$ since $A_{21} \geq A_{12}$.

That is $K_2 \geq K_1$.

Also, $K_3 = A_{11} + A_{21} + A_{31} - A_{12} - A_{22}$

$= A_{11} + (A_{21} - A_{12}) + (A_{31} - A_{22})$

$= K_2 + (A_{31} - A_{22})$

$\geq K_2$ since $A_{31} \geq A_{22}$

Therefore, we have $K_3 \geq K_2 \geq K_1$

Continuing like this, we can prove that $K_1 \leq K_2 \leq$---------$\leq K_n$

Thus it follows that K_1, K_2, ----------, K_n is a monotonically increasing sequence.

Corollary: If $A_{i1} \geq A_{j2}$ for each i and j then the total elapsed time for jobs is same for all the possible sequences.

Proof: The total elapsed time

$T(\sigma) = \sum_{i=1}^{n} A + A_{n2}$

$= \sum_{i=1}^{n} A + (\sum_{i=1}^{n} A - \sum_{i=1}^{n-1} A)$

$= \sum_{i=1}^{n} A + (\sum_{i=1}^{n} A - \sum_{i=1}^{n-1} A)$

$= \sum_{i=1}^{n} A + K_n$

Therefore the total elapsed time for jobs is same for all the possible sequences.

PROBLEM FORMULATION

Let n- jobs (i = 1, 2, ------, n) be processed on two machines M_j (j = 1, 2) in the order M_1M_2. Let a_{ij} be the processing time and s_{ij} be the setup time of i^{th} job on j^{th} machine with probabilities p_{ij} and q_{ij} respectively such that $0 \leq p_{ij} \leq 1$, $\sum_{i=1}^{n} p_{ij} = 1$, $0 \leq q_{ij} \leq 1$, $\sum_{i=1}^{n} q_{ij} = 1$. Let A_{ij} & S_{ij} be the expected processing time and set up time respectively of i^{th} job on j^{th} machine. Let $T_{i,1\to 2}$ be the transportation time of i^{th} job from machine M_1 to machine M_2. The mathematical model of the problem in matrix form is given in table-1.

Consider two job blocks α and β such that the job block α consist of s jobs with fixed order of jobs and β consist of p jobs in which order of jobs is arbitrary such that s + p = n and $\alpha \cap \beta = \emptyset$ i.e. the two job blocks α and β form a disjoint set in the sense that the two blocks have no job in common. Let S = (α, β). Our aim is to find an optimal string S' of job blocks α and β_k i.e. to find an optimal sequence σ_k of jobs which minimizes the elapsed time and hence minimizes the utilization times of machines given that S = (α, β).

Mathematically, the problem is stated as:

Minimize T (σ_k) and hence

Minimize U_2 (σ_k), given that S = (α, β).

PROPOSED ALGORITHM

Step 1: Calculate the expected processing times A_{ij} given by $A_{ij} = a_{ij} \times p_{ij}$.

Step 2: Compute the expected flow times A'_{i1} and A'_{i2} for respective machines M_1 and M_2 as:

$A'_{i1} = A_{i1} - S_{i2}$, and

$A'_{i2} = A_{i2} - S_{i1.}$

Step 3: Compute the processing times G_i and H_i for respective machines M_1 and M_2 as:

$G_i = A'_{i1} + T_{i,1\rightarrow 2}$ and

$H_i = A + T_{i,1\rightarrow 2}$

Step 4: Take equivalent job α for the job block (r, m) and calculate the processing times

G_α and H_α on the guidelines of Maggu and Das [10] as follows:

$G_\alpha = G_r + G_m - \min(G_m, H_r)$

$H_\alpha = H_r + H_m - \min(G_m, H_r)$

If a job block has three or more than three jobs then to find the expected flow times we use the property that the equivalent job for a job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$.

Step 5: Check the structural conditions that $min_i\{G_i\} > max_i\{H_i\}$ or $max_i\{G_i\} \leq min_i\{H_i\}$ for the job block β. If the structural conditions hold good obtain the new job block β_k having jobs in an optimal order from the job block β (disjoint from job block α) by treating job block β as sub flow shop scheduling problem of the main problem. For finding β_k follow the following steps:

(A): Obtain the job J_1 (say) having maximum processing time on 1st machine and job J_r (say) having minimum processing time on 2nd machine. If $J_1 \neq J_r$ then put J_1 on the first position and J_r at the last position and go to 5(C) otherwise go to 5(B).

(B): Take the difference of processing time of job J_1 on M_1 from job J_2 (say) having next maximum processing time on machine M_1. Call this difference as G_1'. Also take the difference of processing time of job J_r on machine M_2 from job J_{r-1} (say) having next minimum processing time on M_2. Call this difference as G_2'. If $G_1' \leq G_2'$ then put J_r on the last position and J_2 on the first position otherwise put J_1 on 1st position and J_{r-1} on the last position. Now follow step 5(C).

(C): Arrange the remaining (p - 2) jobs, if any between 1st job J_1(or J_2) & last job J_r (or J_{r-1}) in any order; thereby due to structural conditions we get the job blocks $\beta_1, \beta_2 \ldots \beta_m$, where m= (p – 2)!; with jobs in optimal order and each having same elapsed time. Let $\beta_k = \beta_1$ (say).

Step 6: Obtain the processing times G_{β_k} and H_{β_k} for the job block β_k on the guidelines of Maggu and Das [10] as defined in step 4. Now, reduce the given problem to a new problem by replacing s-jobs by job block α with expected flow times G_α and H_α and remaining p-jobs by a disjoint job block β_k with expected flow times G_{β_k} and H_{β_k}. The new reduced problem can be represented as in table-2.

Step 7: Check the structural conditions $min_i\{G_i\} > max_i\{H_i\}$ or $max_i\{G_i\} \leq min_i\{H_i\}$ for each job *i* and *k*. If the structural conditions hold good go to Step 8 to find S' otherwise modify the problem.

Step 8: For finding optimal string S' follow the following steps:

(a) Obtain the job I_1 (say) having maximum processing time on 1st machine and job I'_1 (say) having minimum processing time on 2nd machine. If $I_1 \neq I'_1$ then put I_1 on the first position and I'_1 at last position to obtain S' otherwise go to step 8(b).

(b) Take the difference of processing time of job I_1 on M_1 from job I_2 (say) having next maximum processing time on machine M_1. Call this difference as H'_1. Also take the difference of processing time of job I'_1 on machine M_2 from job I'_2 (say) having next minimum processing time on M_2. Call this difference as H'_2. If $H'_1 \leq H'_2$ then put I'_1 on the second position and I_2 at the first position otherwise put I_1 on first position and I'_2 at the second position to obtain the optimal string S'.

Step 9: Compute the in - out table for sequence σ_k of jobs in the optimal string S'.

Step 10: Compute the total elapsed time T (σ_k).

Step 11: Calculate the utilization time U_2 of 2nd machine for optimal sequence σ_k, given by

$$U_2(\sigma_k) = T(\sigma_k) - A_{11}(\sigma_k) - T_{1,1\rightarrow 2}$$

NUMERICAL ILLUSTRATION

To minimize the utilization time for six jobs to be processed in a string of disjoint blocks on two machines as job block α = (2, 5) with fixed order of jobs and job block β = (1, 3, 4, 6) with arbitrary order of jobs such that α ∩ β =Ø. The processing times and setup times with respective probabilities are given in table-3.

Solution: Step 1: The expected processing times and expected setup times for machines M_1 and M_2 are calculated in table-4.

Step 2: The expected flow times for machines M_1 and M_2 are computed in table-5.

Step 3: The processing times $G_i = A'_{i1} + T_{1,1\to 2}$ and $H_i = A'_{i2} + T_{1,1\to 2}$ for machines M_1 and M_2 are given in table-6.

Step 4: The processing times G_α and H_α for the job-block α = (2, 5) are calculated on the guidelines of Maggu and Das [10] as follows:

$G_\alpha = G_r + G_m - \min(G_m, H_r)$ (Here r = 2 & m = 5)

$= 12.7 + 7.6 - \min(7.6, 4.6)$

$= 20.3 - 4.6 = 15.7$

$H_\alpha = H_r + H_m - \min(G_m, H_r)$

$= 4.6 + 3.0 - \min(7.6, 4.6)$

$= 7.6 - 4.6 = 3.0$

Step 5: The structural conditions $\min_i\{G_i\} \geq \max_i\{H_i\}$ hold good and so using step 5 we get $\beta_k = (3, 1, 4, 6)$.

Step 6: Now, we know that the equivalent job for job-block is associative i.e.

$((i_1, i_2), i_3) = (i_1, (i_2, i_3))$ and so we have, $\beta_k = (3, 1, 4, 6) = ((3, 1), 4, 6) = (\alpha_1, 4, 6) = (\alpha_2, 6)$, where $\alpha_1 = (3, 1)$ and $\alpha_2 = (\alpha_1, 4)$. Therefore, the processing times $G_{\beta k}$ and $H_{\beta k}$ for the job block β_k are calculated as:

$G_{\alpha 1} = 12.0 + 4.9 - \min(4.9, 4.7) = 16.9 - 4.7 = 12.2$

$H_{\alpha 1} = 4.7 + 3.7 - \min(4.9, 4.7) = 8.4 - 4.7 = 3.7$

$G_{\alpha 2} = 12.2 + 8.5 - \min(8.5, 3.7) = 20.7 - 3.7 = 17.0$

$H_{\alpha 2} = 3.7 + 4.9 - \min(8.5, 3.7) = 8.6 - 3.7 = 4.9$

$G_{\beta k} = 17.0 + 6.3 - \min(6.3, 4.9) = 23.3 - 4.9 = 18.4$

$H_{\beta k} = 4.9 + 3.3 - \min(6.3, 4.9) = 8.2 - 4.9 = 3.3$

The reduced problem is defined in table-7.

Step 7: We have $\min_i\{G_i\} \geq \max_i\{H_i\}$ and thus the structural relations hold good.

Step 8: The optimal string S' is given by S' = (β_k, α). Hence, the optimal sequence σ_k of jobs as per string S' is σ_k = 3 – 1 − 4 – 6 – 2 –5.

The in-out table for optimal sequence σ_k is computed in table-8

Therefore, the total elapsed time = T (σ_k) = 42.6 units.

Utilization time of machine $M_2 = U_2(\sigma_k) = (42.6 - 12.4)$ units.

= 30.2 units.

Remarks: If we solve the same problem by Johnson's [1] method by treating job block β as sub flow shop scheduling problem of the main problem we get the new job block β' from the job block β (disjoint from job block α) as β' = (4,3,1,6).

The expected flow time $G_{\beta'}$ and $H_{\beta'}$ for the job block β' = (4,3,1,6) on the guidelines of Maggu and Das [10] are calculated below:

Now, $\beta' = (4, 3, 1, 6) = ((4, 3), 1, 6) = (\alpha', 1, 6) = (\alpha'', 6)$; where $\alpha' = (4, 3)$ and $\alpha'' = (\alpha', 1)$.

$G_{\alpha'} = 8.5 + 12.0 - \min(12.0, 4.9) = 20.5 - 4.9 = 15.6$

$H_{\alpha'} = 4.9 + 4.7 - \min(12.0, 4.9) = 9.6 - 4.9 = 4.7$

$G_{\alpha''} = 15.6 + 4.9 - \min(4.9, 4.7) = 20.5 - 4.7 = 15.8$

$H_{\alpha''} = 4.7 + 3.7 - \min(4.9, 4.7) = 8.4 - 4.7 = 3.7$

$G_{\beta'} = 15.8 + 6.3 - \min(6.3, 3.7) = 22.1 - 3.7 = 18.4$

$H_{\beta'} = 3.7 + 3.3 - \min(6.3, 3.7) = 7.0 - 3.7 = 3.3$

The reduced problem is defined in table-9.

By Johnson's [1] algorithm the optimal string S' is given by S' = (β', α).

Therefore, the optimal sequence σ for the original problem corresponding to optimal string S' is given by σ = 4 – 3 – 1 – 6 – 2 – 5. The in – out flow table for the optimal sequence σ is calculated in table-10.

Therefore, the total elapsed time = T (σ) = 42.6 units.

Utilization time of machine $M_2 = U_2(\sigma) = (42.6 - 8.8)$ units.

= 33.8 units.

DISCUSSION

In this paper we have studied a heuristic method to optimize the utilization time of machines which is applicable in case of two stage specially structured flow shop scheduling problems. The parameters taken into consideration were setup time, transportation time and jobs in a string of disjoint job blocks. The results of the study can be applied by considering other parameters such as weightage of jobs, breakdown interval and may be used in case of three stage specially structured flow shop scheduling problems.

CONCLUSION

The specially structured flow shop scheduling problem we have studied take into account the setup time, transportation time and having jobs in a string of disjoint job blocks. We see that the proposed algorithm optimizes both the make span and the utilization for a specially structured two stage flow shop scheduling problem. If we apply the algorithm proposed by Johnson [1], then from table: 10 we see that the utilization time of machine M_2 is $U_2(\sigma)$ = 33.8 units with make span of 42.6 units. However, if the proposed algorithm is applied the utilization time of machine M_2 as per table: 8 is $U_2(\sigma_k)$ = 30.2 units with the same make span of 42.6 units.

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Conflict of Interest

The authors declare that there is no conflict of interest.

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Table 1: Table for Mathematical Model of the Problem in Matrix Form

Jobs	Machine M_1				Transportation time	Machine M_2			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	$T_{i,1\to 2}$	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	a_{11}	p_{11}	s_{11}	q_{11}	$T_{1,1\to 2}$	a_{12}	p_{12}	s_{12}	q_{12}
2	a_{21}	p_{21}	s_{21}	q_{21}	$T_{2,1\to 2}$	a_{22}	p_{22}	s_{22}	q_{22}
3	a_{31}	p_{31}	s_{31}	q_{31}	$T_{3,1\to 2}$	a_{32}	p_{32}	s_{32}	q_{32}
-	-	-	-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	$T_{n,1\to 2}$	a_{n2}	p_{n2}	s_{n2}	q_{n2}

Table 2: Table for Reduced Problem in Matrix Form

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	G_α	H_α
β_k	G_{β_k}	H_{β_k}

Table 3: Table for processing time and set-up time with associated probabilities including transportation time

Jobs	Machine M_1				Transportation Time	Machine M_2			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	$T_{i,1\to 2}$	a_{i2}	p_{i2}	s_{i2}	q_{i2}
1	25	0.1	3	0.1	3	10	0.1	3	0.2
2	44	0.2	2	0.3	4	6	0.2	1	0.1
3	28	0.3	4	0.2	4	5	0.3	2	0.2
4	29	0.2	3	0.1	3	22	0.1	3	0.1
5	60	0.1	6	0.1	2	8	0.2	2	0.2
6	49	0.1	3	0.2	2	19	0.1	3	0.2

Table 4: Table for expected processing time and set-up time including transportation time

Jobs	Machine M_1		Transportation time	Machine M_2	
i	A_{i1}	S_{i1}	$T_{i,1\to 2}$	A_{i2}	S_{i2}
1	2.5	0.3	3	1.0	0.6
2	8.8	0.6	4	1.2	0.1
3	8.4	0.8	4	1.5	0.4
4	5.8	0.3	3	2.2	0.3
5	6.0	0.6	2	1.6	0.4
6	4.9	0.6	2	1.9	0.6

Table 5: Table for processing times A'_{i1}, A'_{i2} and transportation time

Jobs	Machine M_1	Transportation time	Machine M_2
i	A'_{i1}	$T_{i,1\to 2}$	A'_{i2}
1	1.9	3	0.7
2	8.7	4	0.6
3	8.0	4	0.7
4	5.5	3	1.9
5	5.6	2	1.0
6	4.3	2	1.3

Table 6: Table for processing times G_i, H_i and transportation time

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
1	4.9	3.7
2	12.7	4.6
3	12.0	4.7
4	8.5	4.9
5	7.6	3.0
6	6.3	3.3

Table 7: Table for the new reduced problem

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	15.7	3.0
β_k	18.4	3.3

Table 8: In - Out table for machines as per proposed algorithm

Jobs	Machine M_1	Transportation time	Machine M_2
i	In-Out	$T_{i,1\to 2}$	In-Out
3	0.0 – 8.4	4	12.4 – 13.9
1	9.2 – 11.7	3	14.7 – 15.7
4	12.0 – 17.8	3	20.8 – 23.0
6	18.1 – 23.0	2	25.0 – 26.9
2	23.6 – 32.4	4	36.4 – 37.6
5	33.0 – 39.0	2	41.0 – 42.6

Table 9: Table for reduced problem as per Johnson's Algorithm

Jobs	Machine M_1	Machine M_2
i	G_i	H_i
α	15.7	3.0
β'	18.4	3.3

Table 10: In - Out table for machines as per Johnson's Algorithm

Jobs	Machine M_1	Transportation time	Machine M_2
i	In - Out	$T_{i,1\to 2}$	In - Out
4	0.0 – 5.8	3	8.8 – 11.0
3	6.1 – 14.5	4	18.5 – 20.0
1	15.3 – 17.8	3	20.8 – 21.8
6	18.1 – 23.0	2	25.0 – 26.9
2	23.6 – 32.4	4	36.4 – 37.6
5	33.0 – 39.0	2	41.0 – 42.6