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New Intuitionistic Fuzzy Similarity Measures and Application to Pattern Recognition

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ABSTRACT

Aim: The aim of the paper is to introduce three new intuitionistic fuzzy similarity measures. Methodology: To achieve the goal of this paper, an exponential methodology is used with three different functions. The important properties of the proposed measures are discussed axiomatically.

Results: The applicability and efficiency of these new similarity measures in pattern recognition are demonstrated with illustrative examples. It is shown that the proposed measures are efficient, reasonable and simpler than the existing measures.

Conclusion: The proposed intuitionistic fuzzy similarity measures are consistent for the application point of view in the context of pattern recognition

Key Words: Fuzzy set, Intuitionistic fuzzy set, Intuitionistic fuzzy divergence measure, Intuitionistic fuzzy similarity measure

INTRODUCTION

The notion of fuzzy set introduced by Zadeh [1] has acknowledged a central attention from researchers for its useful applications in various fields such as pattern recognition, image processing, speech recognition, bioinformatics, fuzzy aircraft control, feature selection, decision making, etc. Zadeh [2] presented the concept of entropy, as a measure of uncertainty. In 1972, De Luca and Termini [3] introduced an axiomatic structure of fuzzy entropy measure based on Shannon [4] entropy. In the past decades, literature on fuzzy and intuitionistic fuzzy measures of information and their generalizations are significantly extended by the different researchers [5-23].

The idea of Atanassov's intuitionistic fuzzy sets (IFSs) was first invented by Atanassov [24-27] which found to be well suited to deal with both fuzziness and lack of knowledge or non-specificity. It is noticed that the concept of an IFS is the best alternative approach to define a fuzzy set (FS) in cases where existing information is not enough for the definition of imprecise concepts by mean of a conventional FS. Therefore, the concept of Atanassov IFSs is the generalization of the concept of FSs. In 1993, Gau and Buehrer [28] initiated the concept of vague sets. But, Bustince and Burillo [29] presented that the idea of vague sets was the same to that of Atanassov IFSs. As a very significant content in intuitionistic fuzzy mathematics, the study on the similarity measure between IFSs has established more attention in recent years. Similarity measure between IFSs has been extended by many researchers in last decades. Initially, Dengfeng and Chuntian [30] presented the axiomatic definition of similarity measures between IFSs A and B given by

$$S_D(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left|\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)-\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)\right|^p}{n}}$$

Liang and Shi [31] revealed some counter-intuitive cases resulting from the measures provided by Dengfeng and Chuntian [30] and then presented a number of similarity measures to overcome those cases are as

(i) $$S_e^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\left(\phi_\mu(x_i)+\phi_\nu(x_i)\right)^p}{n}}$$

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(ii) $S_s^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} \left|\left(\psi_{s1}(x_i) + \psi_{s2}(x_i)\right)\right|^p}{n}}$

(iii) $S_h^p(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} (\eta_1(x_i) + \eta_2(x_i) + \eta_3(x_i))^p}{3n}}$

Hung and Yang [32] implemented the Hausdorff distance for developing similarity measure between intuitionistic fuzzy sets. Thereafter, many researchers [33-43], provided the definitions of similarity measures between IFSs.

Although there exist several similarity measures of between IFSs, it is expected to have efficient similarity measures which deal with the aspect of uncertainty, that is, fuzziness and non specificity or lack of knowledge and also deals with the real world problems. Briefly motivated by the above mentioned work, in this paper, we relate the exponential approach on IFSs and propose three similarity measures between two IFSs.

The rest of the paper is organized as follows. Methodology section is devoted to review briefly some well-known concepts related to fuzzy set theory and intuitionistic fuzzy set theory. In results section we introduce three new similarity measure between IFSs with the proof of their validity. The application of the proposed similarity measures in pattern recognition is presented in discussion section. Final section concludes the paper.

METHODOLOGY

We begin by reviewing some well-known concepts related to fuzzy set theory and intuitionistic fuzzy set theory.

Definition 1. *Fuzzy Set (FS)* [1]: A fuzzy set A' defined on a finite universe of discourse $X = (x_1, x_2, ..., x_n)$ is given as:

$$A' = \left\{\left\langle x, \mu_{A'}(x)\right\rangle / x \in X\right\} \tag{1}$$

where $\mu_{A'}: X \to [0,1]$ is the membership function of A'. The membership value $\mu_{A'}(x)$ describes the degree of the belongingness of $x \in X$ in A'. When $\mu_{A'}(x)$ is valued in {0, 1}, it is the characteristic function of a crisp i.e., non-fuzzy set.

Definition 2. *Intuitionistic Fuzzy Set (IFS)* [24-27]: An Atanassov intuitionistic fuzzy set (IFS) $X = (x_1, x_2, ..., x_n$ on a finite universe of discourse $X = (x_1, x_2, ..., x_n)$ is defined as

$$A = \left\{\left\langle x, \mu_A(x), \nu_A(x)\right\rangle / x \in X\right\} \tag{2}$$

where $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ with the condition $0 \le \mu_A + \nu_A \le 1 \ \forall x_i \in X$.

The numbers $\mu_A(x_i), \nu_A(x_i) \in [0,1]$ denote the degree of membership and non-membership of x_i to X, respectively.

For each intuitionistic fuzzy set in X we will call $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$, the intuitionistic index or degree of hesitation of x_i in A. It is obvious that $0 \le \pi_A(x_i) \le 1$ for each $x_i \in X$. For a fuzzy set A' in X, $\pi_A(x_i) = 0$ when $\nu_A(x_i) = 1 - \mu_A(x_i)$. Thus, FSs are the special cases of IFSs.

Atanassov [26] further defined set operations on intuitionistic fuzzy sets as follows:

Let $A, B \in IFS(X)$ given by

$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle / x_i \in X\}$,

$B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle / x_i \in X\}$,

(i) $A \subseteq B$ iff $\mu_A(x_i) \le \mu_B(x_i)$ and $\nu_A(x_i) \ge \nu_B(x_i)$ $A = B$.

(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(iii) $A^c = \{\langle x_i, \nu_A(x_i), \mu_A(x_i)\rangle / x_i \in X\}$.

(iv) $A \cap B = \left\{\left\langle x_i, \min(\mu_A(x_i), \mu_B(x_i)), \max(\nu_A(x_i), \nu_B(x_i))\right\rangle / x_i \in X\right\}$.

(v) $A \cap B = \left\{\left\langle x_i, \min(\mu_A(x_i), \mu_B(x_i)), \max(\nu_A(x_i), \nu_B(x_i))\right\rangle / x_i \in X\right\}$.

Hung and Yang [32], Tan and Chen [44] and Chen and Chang [45] adopt the following properties for the validity of a measure to be an intuitionistic fuzzy similarity measure as:

(i) $0 \le S(A,B) \le 1$

(ii) $A = B$ if and only if $A = B$.

(iii) $S(A,B) = S(B,A)$

(iv) If $A \subseteq B \subseteq C$, $S(A,C) \le S(A,B)$

Then $S(A,C) \le S(A,B)$ and $S(A,C) \le S(B,C)$.

RESULTS

It is well known that distance and similarity measure are dual concepts. Therefore, we may use the distance measure $D_{IFS}^E(A,B)$ introduced by Ohlan [6] to propose the new similarity measures as

$$S_E(A,B) = 1 - D_{IFS}^E(A,B) \tag{3}$$

where

$$D_{IFS}^E(A,B) = \sum_{i=1}^{n}\left[2 - \left(1 - \frac{(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))}{2}\right)e^{\left(\frac{(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))}{2}\right)} - \left(1 + \frac{(\mu_A(x_i) - \mu_B(x_i)) - (\nu_A(x_i) - \nu_B(x_i))}{2}\right)e^{\left(\frac{(\nu_A(x_i) - \nu_B(x_i)) - (\mu_A(x_i) - \mu_B(x_i))}{2}\right)}\right]$$

Thus we may define one of similarity measure between IFSs A and B using the exponential operation as follows:

$$S_e(A,B)=\frac{e^{-D^E_{IFS}(A,B)}-e^{-1}}{1-e^{-1}} \tag{4}$$

On the other hand we may define one more new similarity measure as:

$$S_c(A,B)=\frac{1-D^E_{IFS}(A,B)}{1+D^E_{IFS}(A,B)} \tag{5}$$

Theorem 1. The defined measures (3)-(5) between IFSs B and B are valid measures of similarity between intuitionistic fuzzy sets :

Proof: Let us assume f be a monotonic decreasing function and since $0 \le D^E_{IFS}(A,B) \le 1$,

consequently, we have $f(1) \le f(D^E_{IFS}(A,B)) \le f(0)$ and all the other properties will also hold for a monotonic decreasing function.

Now we have to select a useful and reasonable f for each case.

(i) Let us first assume f as $f(x)=1-x$, then the similarity measure $S_E(A,B)=1-D^E_{IFS}(A,B)$ defined in (3) is well-defined and also satisfies all the properties of a valid measure of similarity between IFSs given in methodology section.

(ii) Now we choose the exponential function $f(x)=e^{-x}$, from which we can say that the measure (4), $f(x)=\frac{1}{1+x}$ is well-defined in view of definition provided above.

(iii) On the other hand, if we choose the function f as $f(x)=\frac{1}{1+x}$, for which the similarity measure (5) $S_c(A,B)=\frac{1-D^E_{IFS}(A,B)}{1+D^E_{IFS}(A,B)}$ is also defined in view of definition already given in methodology section.

Hence, in view of definition of Hung and Yang [32], Tan and Chen [44] and Chen and Chang [45] provided above in Methodology section, the three intuitionistic fuzzy similarity measures (3)-(5) are valid measures of similarity measures.

DISCUSSION

We now demonstrate the efficiency of proposed three intuitionistic fuzzy similarity measures in Pattern Recognition by considering the example of Liu [37], Vlachos and Sergiadis [46], Yen [42] and Farhadinia [47].

Example 1

Let $X=\{x_1,x_2,x_3\}$ and three known patterns P_1, P_2 and P_3 which have classifications C_1, C_2 and C_3 respectively are represented by the following IFSs,

$$P_1=\{(x_1,1.0,0.0),(x_2,0.8,0.0),(x_3,0.7,0.1)\},$$
$$P_2=\{(x_1,0.8,0.1),(x_2,1.0,0.0),(x_3,0.9,0.0)\},$$
$$P_3=\{(x_1,0.6,0.2),(x_2,0.8,0.0),(x_3,1.0,0.0)\},$$

where for $i=1,2,3$

$$Q=\{(x_1,0.5,0.3),(x_2,0.6,0.2),(x_3,0.8,0.1)\}\cdot$$

we have an unknown pattern Q, represented by IFS

$$Q=\{(x_1,0.5,0.3),(x_2,0.6,0.2),(x_3,0.8,0.1)\},$$

our aim here is to classify Q to one of the classes C_1, C_2 and C_3. In order to proceed we use the criteria $k^*=\arg\max_k\{S(P_k,Q)\}$,

Table 1: The Computed Values of Three New Intuitionistic Fuzzy Similarity Measures

Q	P_1	P_2	P_3
$S_E(P_i,Q)$	0.7906	0.8345	0.9269
$S_e(P_i,Q)$	0.7011	0.7587	0.8885
$S_c(P_i,Q)$	0.6537	0.7160	0.8638

From the calculated numerical values of different proposed intuitionistic fuzzy similarity measures from (3) - (5) given in Table 1, it is observed that the pattern Q should be classified to C_3. The results are exactly matching with that obtained in Liu [37], Yen [42] and Farhadinia [47].

Example 2

Given three known patterns P_1, P_2 and P_3 which have classifications C_1, C_2 and C_3 respectively. These are represented by the following IFSs in the universe of discourse $X=\{x_1,x_2,x_3\}$:

$$P_1=\{(x_1,0.2,0.5),(x_2,0.5,0.4),(x_3,0.2,0.4)\},$$
$$P_3=\{(x_1,0.1,0.4),(x_2,0.3,0.5),(x_3,0.7,0.1)\},$$
$$P_3=\{(x_1,0.1,0.4),(x_2,0.3,0.5),(x_3,0.7,0.1)\},$$

where for $i=1,2,3$

$$P_i=\left\{\left\langle x_1,\mu_{P_i}(x_1),\nu_{P_i}(x_1)\right\rangle,\left\langle x_2,\mu_{P_i}(x_2),\nu_{P_i}(x_2)\right\rangle,\left\langle x_3,\mu_{P_i}(x_3),\nu_{P_i}(x_3)\right\rangle\right\}\cdot$$

we have an unknown pattern Q, represented by IFS

$$Q=\{(x_1,0.4,0.5),(x_2,0.3,0.2),(x_3,0.6,0.3)\},$$

our aim is to classify Q to one of the classes C_1, C_2 and C_3. In order to proceed we use the criteria $k^* = \arg\max_k \{S(P_k, Q)\}$,

Table 2: Different Values of Three New Intuitionistic Fuzzy Similarity Measures

Q	P_1	P_3	P_3
$S_E(P_i, Q)$	0.8537	0.8994	0.8896
$S_e(P_i, Q)$	0.7847	0.8486	0.8346
$S_c(P_i, Q)$	0.7447	0.8172	0.8012

From the computed numerical values of different proposed intuitionistic fuzzy similarity measures from (3) - (5) given in Table 2, it is observed that the pattern Q should be classified to C_2. Thus, the proposed intuitionistic fuzzy similarity measures are consistent for the application point of view in the context of pattern recognition.

CONCLUSION

Despite the fact that many similarity measures between IFSs have been developed in past years, still there is a good scope that the better similarity measures can be developed, which will have useful applications in the variety of fields. In this paper, we have proposed three new information-theoretic similarity measures for IFSs with the proof of their validity. The efficiency and consistency of these new similarity measures in the context of pattern recognition are presented with help of examples.

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