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Application of Numerical Methods in Calculating the Depth of Submerged Ball in a RO Water System

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ABSTRACT

Objectives: In this work we use the bisection method through MATLAB for finding the depth of a submerged ball in a RO water tank.

Material and Method: Using MATLAB, and by Bisection method, we find the depth of the submerged ball in a RO water system.

Results: We know that Buoyancy (upthrust) is an upward force exerted by a fluid that opposes the weight of an immersed object. Archimedes' principle indicates that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces and it acts in the upward direction at the centre of mass of the displaced fluid. Using this principle we have formed the polynomial equation, according to the radius of the ball submerged in water. We have solve this polynomial equation using MATLAB, and by Bisection method. This gives us the depth of the submerged ball in a RO water system.

Conclusion: The polynomial equation in x we formed is a nonlinear equation. Solving it would give us the value of ' x ', that is, the depth to which the floatball is submerged under water.

Key Words: MATLAB, Buoyancy force, Archimedes Principle, Bisection method

INTRODUCTION

The main goals of this work is to introduce the concepts of numerical methods and introduce Matlab in an Engineering framework. By this we do not mean that every problem is a "real life"

Engineering application, but more that the engineering way of thinking is emphasized throughout discussion. In a RO water tank we are opening the tank frequently to check the water level means, there is no use of the RO system, because the water get spoiled due to outside dust. So, we have to fix a water level checker. This should maintain the water level without opening the tank. So that we have use Plastic float ball and that automatically start and stop water flow.

The float is adjustable to allow you to control the water height. In the Newest Reverse Osmosis System a Adjustable Mini Float Ball Valve can Shut off the water in the automatic Fill Feed Fish Tank Aquarium RO Water. We have to find the depth of the ball which is submerged in water.

The permeate water then enters the storage tank. The tank level is controlled by a float switch, which turns the RO unit as well as the Spectra guard pump on and off. After the pressure tank, the water is re- pressurized by a pump and sent to the home usage.

In this case we have to decide the water level height and fixing of lid in the tank, for that we should find depth of the ball using the bisection method.

PRELIMINARIES

Definition

Buoyancy (upthrust) is an upward force exerted by a fluid that opposes the weight of an immersed object.

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Definition

The weight of the displaced fluid is directly proportional to the volume of the displaced fluid (if the surrounding fluid is of uniform density). The weight of the object in the fluid is reduced, because of the force acting on it, which is called upthrust.

Archimedes' principle

Archimedes' principle indicates that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces and it acts in the upward direction at the centre of mass of the displaced fluid. Archimedes' principle is a law of physics fundamental to fluid mechanics. It was formulated by Archimedes of Syracuse.

In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid. Thus the pressure at the bottom of a column of fluid is greater than at the top of the column. Similarly, the pressure at the bottom of an object submerged in a fluid is greater than at the top of the object. This pressure difference results in a net upwards force on the object. The magnitude of that force exerted is proportional to that pressure difference, and (as explained by Archimedes' principle) is equivalent to the weight of the fluid that would otherwise occupy the volume of the object, i.e. the displaced fluid.

In on Floating Bodies, Archimedes suggested that, Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.

For a fully submerged object, Archimedes' principle can be reformulated as follows:

Apparent immersed weight = weight of object – weight of displaced fluid

then inserted into the quotient of weights, which has been expanded by the mutual volume

$$\frac{densityofobject}{densityoffluid} = \frac{weight}{weightofdisplacedfluid}$$

yields the formula below. The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes:

$$\frac{densityofobject}{densityoffluid} = \frac{weight}{weight - apparentimmensedweight}$$

Example: If you drop wood into water, buoyancy will keep it afloat.

TO FIND THE DEPTH TO WHICH THE BALL WILL GET SUBMERGED WHEN FLOATING IN WATER

In a RO water tank we are opening the tank frequently to check the water level means, there is no use of the RO system, because the water get spoiled due to outside dust. So, we have to fix a water level checker. This should maintain the water level without opening the tank. So that we have use Plastic float ball and that automatically start and stop water flow.

The float is adjustable to allow you to control the water height. In the newest reverse osmosis system a adjustable mini float ball valve can shut off the water in the automatic fill feed fish tank aquarium RO water. We have to find the depth of the ball which is submerged in water.

The permeate water then enters the storage tank. The tank level is controlled by a float switch, which turns the RO unit as well as the Spectra guard pump on and off. After the pressure tank, the water is re- pressurized by a pump and sent to the home usage.

Suppose A ball has a gravity of 0.8 and has a radius of 6 cm is floating in water. We are trying to find the depth of the ball which the ball will get submerged when floating in water.

According to Newton's third law of motion, every action has an equal and opposite reaction. In this case, the weight of the ball is balanced by the buoyancy force.

$$\text{Weight of ball} = \text{Buoyancy force} \quad (1)$$

The weight of the ball is given by

Weight of ball = (volume of ball) x(Density of ball) x(Acceleration due to gravity)

$$= \left(\frac{4}{3}\pi r^3\right)(\rho_b)(g) \quad (2)$$

Where R = radius of ball(m) , ρ = density of ball (kg/m^3), g = acceleration due to gravity(m/s^2).

The buoyancy force is given by

Buoyancy force = Weight of water displaced

= (Volume of ball under water)(Density of water)(Acceleration due to gravity)

$$\pi x^2\left(R - \frac{X}{3}\right)\rho_w g \quad (3)$$

where x = depth to which ball is submerged,

ρ_w = density of water.

DERIVATION OF THE FORMULA FOR THE VOLUME OF A BALL SUBMERGED UNDER WATER.

How do you find that the volume of the ball submerged under water as given by

$$V = \frac{\pi h^2 (3r - h)}{3} \qquad (4)$$

where

r = radius of the ball,

h = height of the ball to which the ball is submerged.

From calculus

$$V = \int_{r-h}^{r} A dx \qquad (5)$$

where A is the cross-sectioned area at a distance x from the center of the sphere. The lower limit of integration is $x = r - h$ as that is where the water line is

Figure 2 - Deriving the equation for volume of ball under water and the upper limit is r as that is the bottom of the sphere. So, what is the A at any location x .

From Figure 2, for a location x , $OB = x$, $OA = r$,

Then

$$AB = \sqrt{OA^2 - OB^2} = \sqrt{r^2 - x^2}. \qquad (6)$$

and AB is the radius of the area at x . So at location B is

$$A = \pi (AB)^2 = \pi(r^2 - x^2) \qquad (7)$$

So

$$\mathbf{V} = \int_{r-h}^{r} \pi (r^2 - x^2) dx$$

$$= \pi \left(r^2 x - \frac{x^3}{3} \right)_{r-h}^{r}$$

$$= \pi \left(r^2 x - \frac{x^3}{3} \right)_{r-h}^{r}$$

$$= \pi \left(r^2 x - \frac{x^3}{3} \right) - \left(r^2 (r - h) - \frac{(r - h)^3}{3} \right)$$

$$= \frac{\pi h^2 (3r - h)}{3}$$

Now substituting Equations (2) and (3) in Equation (1),

$$\frac{4}{3} \pi R^3 [\rho_b] (g) = \pi x^2 \left(R - \frac{X}{3} \right) \rho_w g$$

$$4 (R^3)(\rho_b) = 3 x^2 (R - x/3) \rho_w$$

$$4 (R^3)(\rho_b) - 3 x^2 R \rho_w + x^3 \rho_w = 0$$

Divide by ρ_w

$$4 (R^3) \frac{\rho_b}{\rho_w} - 3 x^2 R + x^3 = 0$$

$$4 (R^3) \Upsilon_b - 3 x^2 R + x^3 = 0 \text{ ------------------(8)}$$

Where the specific gravity γ_b of the ball is given by

$$\gamma_b = \frac{\rho_b}{\rho_w} \qquad (9)$$

γ_b = 0.8cm = 0.008m

R = 6 cm = 0.06m

Substituting in equation (4), we get

$4(0.06)^3(0.008) - 3 x^2(0.06) + x^3 = 0$

$691 \times 10^{-6} - 0.18 x^2 + x^3 = 0$

This equation is a nonlinear equation. Solving it would give us the value of ‘ x ’, that is, the depth to which the floatball is submerged under water.

The equation that gives the depth x to which the ball is submerged under water is given by

SOLUTION USING BISECTION METHOD

Use the bisection method of finding roots of equations to find the depth x to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration, and the number of significant digits at least correct at the end of each iteration.

From the physics of the problem, the ball would be submerged between $x = 0$ and $x = 2R$, where

R = radius of the ball, that is

$0 \le x \le 2R$

$0 \le x \le 2(0.06)$ $0 \le x \le 0.12$

Solution:

Let us assume that the root of $x^3 - 0.18x^2 + 691 \times 10^{-6} = 0$ lies between [0,0.12]

We find the solution using bisection method

Here we are going to use the MatLab for solving the same polynomial equation. In this procedure we have find the solution with a graph easily.

MATLAB in Algorithms:

```
>> % Bisection method
% Find the root by bisection method
f= @(x)x^3 - (0.18)*x^2 +(691) * 10^-6;
a=0;
b=0.12;
for i=1:100
  c=(a+b)/2;
if f(c)>0
b=c;
else a=c;
end
end
a=0.05; b=0.12; p=c;
for i=1:100
c=(a+b)/2;
er(i)=f(c)-f(p);
if f(c)>0
b=c;
else a=c;
end
end
fprintf('Root of given equation is %f',c)
plot(er);
title('Plot of error')
xlabel('Number of iterations')
ylabel('Error')
Root of given equation is 0.0850000>>
```

Table 1: Below table contains ten iterations of the function.

Iterations	m	n	T=(m+n)/2
1	0	0.1	0.05
2	0.05	0.1	0.075
3	0.075	0.1	0.0875
4	0.075	0.0875	0.0813
5	0.0813	0.0875	0.0844
6	0.0844	0.0875	0.08595
7	0.0844	0.08595	0.08518
8	0.08518	0.08595	0.0856
9	0.08518	0.0856	0.08539
10	0.08539	0.0856	0.0855

CONCLUSION

We find the depth of the submerged ball into the water using the buoyancy force and by the bisection method, Matlab is very helpful on finding the solution. The polynomial equation in x we formed is a nonlinear equation. Solving it would give us the value of ' *x* ', that is, the depth to which the float-ball is submerged under water.

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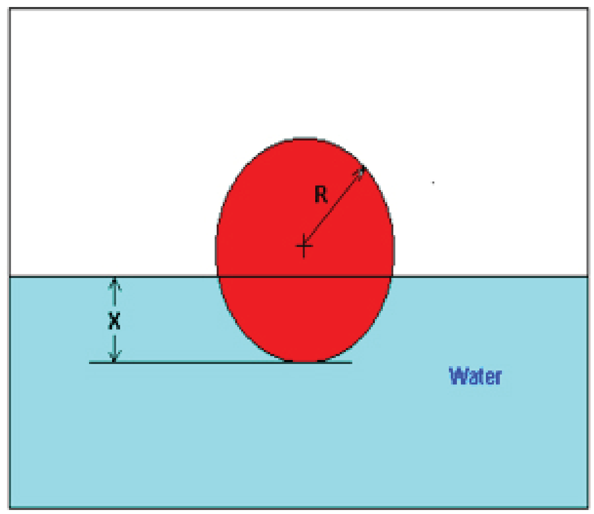

Figure 1: Depth to which the ball is submerged in water.

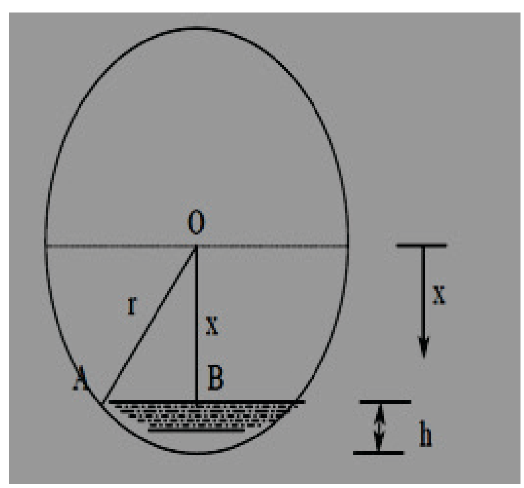

Figure 2: Deriving the equation for volume of ball under water and the upper limit is *r* as that is the bottom of the sphere. So, what is the A at any location *x* .

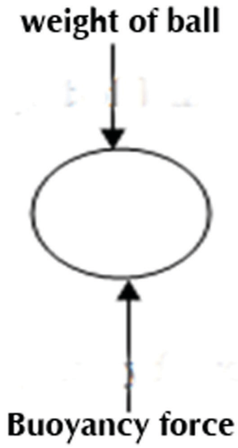

Figure 3: Free Body Diagram showing the forces acting on the ball all immersed in water.

Graph in MATLAB: