Research Article

IJCRR
Section: Healthcare
Sci. Journal
Impact Factor
4.016

SOLUTION OF 3-DIMENSIONAL WAVE EQUATION BY METHOD OF SEPARATION OF VARIABLES

Rajan Singh, Mukesh Chandra, B.K. Singh

Department of Mathematics, I.F.T.M. University, Moradabad, 244001, UP, India

ABSTRACT

We study solutions of the 3-Dimensional wave equation with boundary Conditions on Cartesian co-ordinates, and we also study the analogous problem on a certain axis. This gives an alternative method of obtaining solutions of a corresponding problem in 3-Dimensional wave equation further, In this paper, we find the solution of 3-D wave equation .Three dimensional wave occur in earth quake, tsunami and many physical states. In this paper we discussed the 3-D wave equation in XYZ axis and using partial differential equation.

Key Words: 3-D wave equation, Partial differential Equation, Fourier series.

INTRODUCTION

Solutions of the wave equation with boundary conditions have many practical applications in engineering and physics. The paradigm of such textbook problems is that describing vibrations of a circular membrane (the shape of a drum) requiring solutions of the wave equation in a 3- dimensional. These solutions must vanish on the rectangular boundary of the membrane [8]. A theoretical application of much current interest, requiring such solutions, is the computation of sound energies for spherical boundary conditions. We show here that it is just as easy to set up such problems in a certain co-ordinate plane [3]

RESEARCH METHODOLOGY

The following Research Methodology is adopted for the proposed Research paper:

- Identification of the problem
- Collection and study of related literature
- Mathematical formulation of the problem
- Analysis and numerical solution of the mathematical model
- Interpretation of results
- Conclusion

Mathematical formulation of the problem

The physical setting for our problem is as follows. We consider the three dimensional wave equations with the normal axis.

Three dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = \rho^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{1.1}$$

$$\text{Let } u(x,y,z,t) = XYZT \tag{1.2}$$

Where X is function of x only, Y is function of y only, Z is function of z only and T is function of t only.

$$\frac{\partial^2 u}{\partial t^2} = XYZT'', \frac{\partial^2 u}{\partial x^2} = X''YZT, \frac{\partial^2 u}{\partial y^2} = XY''ZT, \frac{\partial^2 u}{\partial z^2} = XYZ''T$$

$$\frac{\partial^2 u}{\partial t^2} = XYZT'' \tag{1.3}$$

From equation (1.1)

We have,

$$\frac{1}{\rho^2} XYZT'' = X''YZT + XY''ZT + XYZ''T$$

Or

Corresponding Author:
Rajan Singh, Department of Mathematics, I.F.T.M. University, Moradabad, 244001, UP, India.
E-mail: rajan0779@rediffmail.com, mukesh.111977@gmail.com

Received: 09.05.2015 **Revised:** 28.05.2015 **Accepted:** 25.06.2015

$$\frac{1}{\rho^2}\frac{T''}{T}=\frac{X}{X}+\frac{Y''}{Y}+\frac{Z''}{Z} \tag{1.4}$$

This will be true only when each member is a constant,

$\frac{d^2X}{dx^2}+k_1^2X=0,\frac{d^2Y}{dy^2}+k_2^2Y=0,\frac{d^2Z}{dz^2}+k_3^2X=0$ and

$$\frac{d^2T}{dt^2}+(k_1^2+k_2^2+k_3^2)T=0 \tag{1.5}$$

The solution of equation (1.5) are

$X=c_1\cos k_1x+c_2\sin k_1x$, $Z=c_5\cos k_3z+c_6\sin k_3z$ and

$$T=c_7\cos\sqrt{(k_1+k_2+k_3)}\,\rho t+c_8\sin\sqrt{(k_1+k_2+k_3)}\,\rho t \tag{1.6}$$

The solution of equation (1.1)

$(c_1\cos k_1x+c_2\sin k_1x)$ $(c_1\cos k_1x+c_2\sin k_1x)$

$(c_5\cos k_3z+c_6\sin k_3z)$ $(c_5\cos k_3z+c_6\sin k_3z)$

$$(c_7\cos\sqrt{(k_1+k_2+k_3)}\,\rho t+c_8\sin\sqrt{(k_1+k_2+k_3)}\,\rho t) \tag{1.7}$$

Now Let us suppose that membrane is cuboids and stretched among the lines $x=0,x=a;y=a,y=b;z=c,z=c$

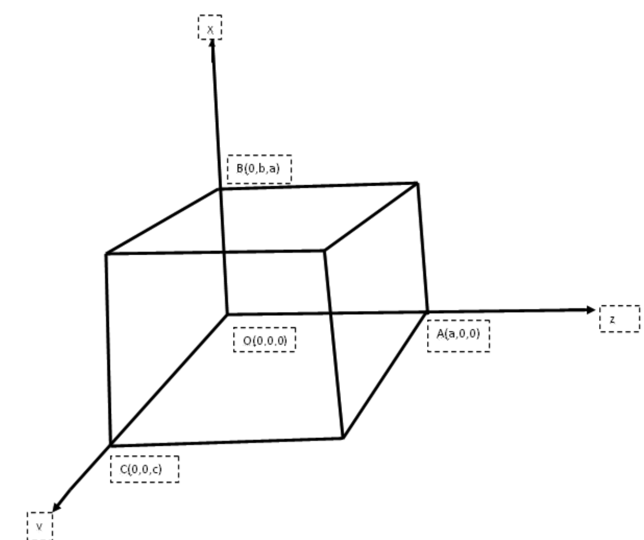

Figure 1: 3-dimensional wave form

Then the boundary value conditions are:

$(i)u(0,y,z,t)=0, when\ x=0\ (ii)u(a,y,z,t)=0, when\ x=a\ (iii)u(x,0,z,t)=0, when\ y=0$
$(iv)u(x,b,z,t)=0, when\ y=b\ (v)u(x,y,0,t)=0, when\ z=0\ (vi)u(x,y,c,t)=0, when\ y=c$
$for\ all\ t.$

Applying condition (i) and (ii)

We have $c_1=0\ and\ k_1=\frac{l\pi}{a}$

$$u(x,y,z,t)=c_2\sin\tfrac{l\pi x}{a}(c_3\cos k_2y+c_4\sin k_2y)(c_5\cos k_3z+c_6\sin k_3z)$$

$$(c_7\cos\sqrt{(k_1+k_2+k_3)}\,\rho t+c_8\sin\sqrt{(k_1+k_2+k_3)}\,\rho t) \tag{1.8}$$

Applying condition (iii) and (iv)

We have $c_3=0\ and\ k_2=\frac{m\pi}{b}$

$$u(x,y,z,t)=c_2c_4\sin\tfrac{l\pi x}{a}\sin\frac{m\pi y}{b}(c_5\cos k_3z+c_6\sin k_3z)$$

$$(c_7\cos\sqrt{(k_1+k_2+k_3)}\,\rho t+c_8\sin\sqrt{(k_1+k_2+k_3)}\,\rho t) \tag{1.9}$$

Now Applying condition (v) and (vi)

We have $c_5=0\ and\ k_3=\frac{n\pi}{c}$

$$u(x,y,z,t)=c_2c_4c_6\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}$$

$$(c_7\cos\sqrt{(k_1+k_2+k_3)}\,\rho t+c_8\sin\sqrt{(k_1+k_2+k_3)}\,\rho t) \tag{1.10}$$

$$u(x,y,z,t)=c_2c_4c_6\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}$$

$$\left(c_7\cos\sqrt{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}\,\pi\rho t+c_8\sin\sqrt{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}\,\pi\rho t\right) \tag{1.11}$$

Or

$$u(x,y,z,t)=c_2c_4c_6\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}(c_7\cos pt+c_8\sin pt) \tag{1.12}$$

Where

$$p=\sqrt{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}\,\pi\rho$$

General solution as

$$u(x,y,z,t)=\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{=1}^{\infty}\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}\left(A_{lmn}\cos pt+B_{lmn}\sin pt\right) \tag{1.13}$$

Suppose the membranes start from the rest from the initial position $u(x,y,z,0)=f(x,y,z)$

If $B_{lmn}=0$, we get $B_{lmn}=0$

$$f(x,y,z,)=\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{=1}^{\infty}A_{lmn}\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}\cos pt \tag{1.14}$$

Using Fourier series,

$$\int_0^a\int_0^b\int_0^c f(x,y,z,)\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}\,dx\,dy\,dz=\frac{abc}{8}A_{lmn}$$

Or

$$A_{lmn}=\frac{8}{abc}\int_0^a\int_0^b\int_0^c f(x,y,z,)\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}\,dx\,dy\,dz$$

Thus the required solution of 3-D wave equation is

$$f(x,y,z,)=\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{=1}^{\infty}A_{lmn}\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}\cos\left(\sqrt{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}\right)\pi\rho t \quad (1.15)$$

Where

$$A_{lmn}=\frac{8}{abc}\int_0^a\int_0^b\int_0^c f(x,y,z,)\sin\frac{l\pi x}{a}\sin\frac{m\pi y}{b}\sin\frac{n\pi c}{z}\,dx\,dy\,dz$$

CONCLUSION

In this paper, for a general solution of three dimensional of wave equations is fond and with the help of this solution, we have to find varies kind of solution wave equations for example radio waves, telephonic wave etc.

ACKNOWLEDGEMENT

Authors acknowledge the immense help received from the scholars whose articles are cited and included in the manuscript. The authors are also grateful to authors / editors / publishers of all those articles, journals and books from where the literature for this article has been reviewed and discussed. Authors are grateful to IJCRR editorial board members and reviewers for their useful comments that lead the improvement of the manuscript.

REFERENCES

1. Adomian, G. (1994)., Solving Frontier problems of Physics: The decomposition method. Kluwer Academic Publishers.
2. Courant, R. & Hilbert, D. (1962), Methods of Mathematical Physics II, New York: Wiley-Interscience.
3. Evans, L. C. (1998), Partial Differential Equations, Providence: American Mathematical Society, ISBN 0-8218-0772-2.
4. Ibragimov, Nail H (1993), CRC Handbook of Lie Group Analysis of Differential Equations Vol. 1-3, Providence: CRC-Press, ISBN 0-8493-4488-3.
5. John, F. (1982), Partial Differential Equations (4th ed.), New York: Springer-Verlag, ISBN 0-387-90609-6.
6. Jost, J. (2002), Partial Differential Equations, New York: Springer-Verlag, ISBN 0-387-95428-7.
7. Lewy, Hans (1957), "An example of a smooth linear partial differential equation without solution", Annals of Mathematics. Second Series 66 (1): 155–158.
8. Polyanin, A. D. (2002), Handbook of Linear Partial Differential Equations for Engineers and Scientists, Boca Raton: Chapman and Hall/CRC Press, ISBN 1-58488-299-9.
9. Wazwaz, Abdul-Majid (2009). Partial Differential Equations and Solitary Waves Theory. Higher Education Press. ISBN 90-5809-369-7.
10. Zwillinger, D. (1997), Handbook of Differential Equations (3rd ed.), Boston: Academic Press, ISBN 0-12-784395-7.