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APPLICATION OF GRAFTED POLYNOMIAL MODEL AS APPROXIMATING FUNCTIONS IN FORECASTING COTTON PRODUCTION TRENDS IN ZAMFARA STATE, NIGERIA[1995-2013].

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ABSTRACT

The aim of this paper was to predict cotton production trends based on time series data from 1995 to 2013 in Zamfara state, Nigeria. The forecasting model that was applied to estimate cotton production trends was a grafted polynomial model "quadratic-quadratic-linear" function. Grafted models are used in econometrics to embark on economic analysis involving time series. Economic time series data were used in order to estimate the possible production and supply trends of cotton in the study area. In the application of the grafted polynomial model used to forecast cotton production trends, the estimates of the linear and grafted functions were utilized to obtain ex-post forecast of cotton. The values of the grafted (mean) function of 123,000 tons were closer to the observed values of 129,000 tons resulting in smaller difference during the sub- period (2006-2013) under consideration when compared with linear values of 137,000 tons. The forecast of production and supply trends among cotton farmers revealed that the grafted model provided better estimates since they were closer to the observed values during the sub-period under consideration.

Key Words: Grafted model, Forecasting, Cotton, Production, Zamfara state, Nigeria

INTRODUCTION

At independence, the contribution of agriculture to the GDP was about 25% between 1975 and 1977. This was partly due to the phenomenal growth of the mining and partly as a result of the disincentives created by macroeconomic environment. Similarly, the growth rate of agricultural productivity exhibited a downward trend during the period. Thus, between 1970 and 1982 agricultural productivity stagnated at less than one percent annual growth rate at a time when the population growth rate was 2.5 to 3.0% per annum (Adubi, 2001).

According to the National Bureau of Statistics (NBS) (2011), the *percent* share in the GDP of the crop sub-sector between 1981 to 1990 had been fluctuating between 28.37% and 22.99% and did not register any significant increase. This trend continued as the contribution of the crop sub-sector was almost stagnant at about 36% from 1994 to 1997 and from 2003 to 2006.

Furthermore, the Central Bank of Nigeria(CBN, 2011) in its annual report indicated the *per cent* share in total of the contribution of the agricultural sector to the GDP at 1990 constant basic prices. From 2007 through 2012, the share has been declining from 42% of the total GDP to 40.2%. The place of the crop production sub-sector in the total GDP have shown similar trend with a decline from 37.5% to 35.8% between the same periods. Despite these marginal decline in recent years, the demand for many agricultural products outweighs the supply.

It is with respect to this that cotton was chosen to form the basis of this study. With regards to fibre crop, cotton is an important crop in the world, it ranks first followed by jute,

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kenaf and sisal in the world production of fibres. It is noticeable from the performance of the cotton production industry that since 2003/2004 cropping season, there has been a fall and fluctuating pattern in the production trends in cotton. According to United State Department of Agriculture, (USDA, 2011), the production trend in cotton had not witnessed remarkable improvement between 2007/2008 cropping year while the 2010 – 2012 cropping seasons experienced a decline.

This phenomenon revealed a glaring disparity between demand and supply thereby creating a gap in the cotton production industry. Batterham (2000) asserted that supply is yet to satisfy the level of demand for cotton. This has caused great concern in the textile cotton fibre supply situation in the local market and export profile in the country thereby having a declining effect in its contribution to the agricultural economy of the country.

Research Question and Objectives

It is based on the above credence that these research question and objective were raised.

What are the patterns of production and supply trends for cotton in the state?

The objective of the study is to;

Predict cotton production trends based on time series data from 1995 to 2013 in the state.

METHODOLOGY

Study Area

Zamfara state was used for this study. The state lies between latitude 10^0 50`N and $13^0$ 38`N and longitudes 4^o16`E and $7^o$18`E. The state is located in the Sudan Savanna ecological zone of Nigeria. It has a land area of 39,762km^2. Zamfara state shares common borders with Sokoto and the Republic of Niger to the north, Katsina and Kaduna states to the east, Niger and Kebbi states in the South (Yakubu, 2005, www.zamfarastate.net, 2010). The state has a population of about 3,259,846 people in 2006 according to the National Population Commission (NPC; 2006). This is projected in 2011 to be 3,667,326 representing3.2% annual growth rate in population (UNFPA 2013).The climate is essentially that of tropical climate. The climate is generally characterized by alternating dry and wet seasons.

Data Source

Secondary data were collected and used in this study. Data on cotton output from the state for various years were collected from Central Bank of Nigeria (CBN annual reports), Food and Agriculture Organizations (FAO) production yearbook or quarterly bulletin of statistics, Federal Office of Statistics (FOS), National Agricultural Extension and Research Liaison Services (NAERLS) Wet Season Production Report, Bulletin of Prices, internet and the Zamfara State Agricultural Development Project (ZADP).

Grafted Polynomial Model

Grafted polynomial model was used to achieve the objective of the study. Grafted models are used in econometrics to embark on economic analysis involving time series. It was assumed that different functional forms may fit different segments of a time series or response studies. Segments of polynomials can be used to approximate production surfaces or frontiers and to forecast time series. The segment to be used to forecast time series as in trend studies must end in a linear form.These segmented curves are restricted to be continuous and differentiable at the joined points. This is shown in appendix I

The Model

We attempted to improve upon the linear model represented by equation (1) by dividing the available time series into segments. A preliminary graphical representation of the observed time series data in appendix I indicates threetime segments and the quadratic-quadratic-linear trend function were hence suggested. From this three time segments, a mean (grafted) functional model was derived in which the objective is to entrench and absorb all the key local trends in the time series Q_t. The mean function in equation (15) required for the forecasts was obtained by substituting for a_o, a_1,b_o and b_1 in equations (1)-(3). This was summarised in appendix (II).

This mean function is now continuous, differentiable and linear in all parameters and structural coefficients.The general equation of the linear trend model used in forecasting along with the mean function is of the general form;

$$= a + bt \quad (1)$$

This is an observed time series that do not relate linearly to trend as shown in the graph above. The three segments of the functional relationship can be expressed as functions of the form;

$$= a_0 + a_1t + a_2t^2, \text{ for } t \leq k_1 \quad (2)$$

$$= b_0 + b_1t + b_2t^2, \text{ for } k_1 < t \leq k_2 \quad (3)$$

$$= c_0 + c_1t, \text{ for } t > k_2 \quad (4)$$

There should be *a priori* expectation. In this model, we are assuming that the mean function is to be continuous, linear in parameters and differentiable at the joined points (k_1 and

k_2) as contended by Fuller, (1969); and Philip, (1990). In other words, we need to derive a mean function which encompasses all the key local trends observed in the time series . Thus, it is imperative that the following restrictions must hold. There are four restrictions namely; two continuities and two differentiabilities.

At continuity, the following expression holds.

$$a_0 + a_1k_1 + a_2k_2^2 = b_0 + b_1k_1 + b_2k_1^2 \quad (5)$$

$$b_0 + b_1k_2 + b_2k_2^2 = c_0 + c_1k_2 \quad (6)$$

At differentiability, we derive these expressions as shown in equations (7) and (8).

$$a_1 + 2a_2k_2 = b_1 + 2b_2k_1 \quad (7)$$

$$b_1 + 2b_2k_2 = c_1 \quad (8)$$

It is noticeable that, there are eight parameters namely a_0, a_1, a_2, b_0, b_1, b_2, c_0, and c_1. There are also four restrictions on the mean function. In other words, this implies that only four parameters can be estimated. The parameters to be estimated depend upon the motive of generating or formulating the mean function which is to forecast.

For forecasting, it is highly important to retain the coefficient in the terminal trend function and this is usually linear. Thus c_0, c_1, a_2 and b_2 are to be retained for subsequent estimation while a_0, a_1, b_0 and b_1 are to be dropped and eliminated.

For ease and simplicity, it is better to start the derivations with equation (8) and making b_1 the subject of the formular. From equation (8).

$$b_1 + 2b_2k_2 = c_1$$

$$b_1 = c_1 - 2b_2k_2 \quad (9)$$

From equation (7)

$$a_1 + 2a_2k_1 = b_1 + 2b_2k_1$$

$$a_1 = b_1 + 2b_2k_1 - 2a_2k_1$$

Substitute for b_1

$$a_1 = c_1 - 2b_2k_2 + 2b_2k_1 - 2a_2k_1$$

$$a_1 = c_1 - 2b_2(k_2 - k_1) - 2a_2k_1 \quad (10)$$

From equation (6)

$$b_0 + b_1k_2 + b_2k_2^2 = c_0 + c_1k_2$$

Substituting for b_1

$$b_0 + (c_1 - 2b_2k_2)k_2 + b_2k_2^2 = c_0 + c_1k_2$$

$$b_0 + c_1k_2 - 2b_2k_2^2 + b_2k_2^2 = c_0 + c_1k_2$$

$$b_0 = c_0 + c_1k_2 - c_1k_2 + 2b_2k_2^2 - b_2k_2^2$$

$$b_0 = c_0 + b_2k_2^2 \quad (11)$$

From equation (5),

$$a_0 + a_1k_1 + a_2k_1^2 = b_0 + b_1k_1 + b_2k_1^2$$

Substituting for a_1, b_0 and b_1

$$a_0 + [c_1 - 2b_2k_2 + 2b_2k_1 - 2a_2k_1]k_1 + a_2k_1^2 = c_0 + b_2k_2^2 + (c_1 - 2b_2k_2)k_1 + b_2k_1^2$$

$$a_0 + c_1k_1 - 2b_2k_2k_1 + 2b_2k_1^2 - 2a_2k_1^2 + a_2k_1^2 = c_0 + b_2k_2^2 + c_1k_1 - 2b_2k_2k_1 + b_2k_1^2$$

$$a_0 + b_2k_1^2 - a_2k_1^2 = c_0 + b_2k_2^2$$

$$a_0 = c_0 + b_2k_2^2 - b_2k_1^2 + a_2k_1^2$$

$$= c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2$$

$$a_0 = c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2 \quad (12)$$

From equation (4)

$$= a_0 + a_1t + a_2t^2$$

Substituting for a_0 *and* a_1

$$= [c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2] + [c_1 - 2b_2(k_2 - k_1) - 2a_2k_1]t + a_2t^2$$

$$= c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2 + [c_1 - 2b_2k_2 + 2b_2k_1 - 2a_2k_1]t + a_2t^2$$

$$= c_0 + b_2k_2^2 - b_2k_1^2 + a_2k_1^2 + c_1t - 2b_2k_2t + 2b_2k_1t - 2a_2k_1t + a_2t^2$$

$$= c_0 + c_1t + a_2t_1^2 - 2a_2k_1t + a_2k_1^2 + b_2k_2^2 - b_2k_1^2 - 2b_2k_2t + 2b_2k_1t$$

$$= c_0 + c_1t + a_2(t_1 - 2k_1t + k_2) + b_2(k_2^2 - k_1^2 - 2k_2t + 2k_1t)$$

$$= c_0 + c_1t + a_2(t - k)^2 + b_2[(k_2^2 - k_1^2) - 2(k_2 - k_1)t] \quad (13)$$

Equation can be transformed as

$$= c_0x_0 + c_1x_1 + a_2x_2 + b_2x_3 \quad (14)$$

For simplicity, equation (14) can be represented as;

……………………………………………… (15)

Where;

$x_0 = 1$; for all t, $x_1 = t$; for all t

$x_2 =$; for k t

= 0 otherwise

$x_3 = (k_2^2 - k_1^2) - 2(k_2 - k_1)$ t for k_1 t

$= (t - k_2)$; for $k_1 < t \leq k_2$

$= 0$

Equation (15) represents a grafted continuous (mean) function which encompasses all the key local trends indicated by the set of restrictions in equations (2), (3) and (4). Equation (15) is the empirical (grafted) model suitable for computing an ordinary least Squares (OLS) regression.

RESULTS AND DISCUSSION

Forecasting Cotton Production Trends in Zamfara State, Nigeria

The linear function and the mean functions represented in equations (1) and (15) were utilized in the analysis using the ordinary least squares (OLS) method. The results of the regression analysis and the estimates of the structural parameters are presented in table 1. The data for the 2006-2013 Sub-period were retained for the ex-post evaluation or forecast of the estimated equation. Also, equation (1) was estimated in order to purposely evaluate the predictive performance of equation (15).

Table 1: Estimates of the Structural Parameters in Predicting Cotton Production Trend in Zamfara State, Nigeria.

Variable	Linear trend	Grafted (Mean)trend
Intercept	168*** (10.12)	314*** (5.79)
X_1	-2 (-1.38)	-12.4*** (-3.25)
X_2	-	0.30 (0.09)
X_3	-	-1.53** (-2.08)
R^2	0.10	0.504
Adjusted R^2	0.048	0.39
df.	18	16
*** $P < 0.01$	**$P < 0.05$	

Figures in parenthesis are the T-values.

The calculated T-values for the grafted function in the trend model for both X_1 and X_3 were statistically significant at 1% and 5% levels of probability. This suggests that the mean trend model must have captured and infused all the observed key and major local trends in the model. Unlike the mean trend model, the linear trend model failed to explain the desired variation observed in cotton production trend. This is because time series do not always correlate linearly to trend during the entire sample period.

The estimates of the linear and grafted functions in table 1 were utilized to obtain ex-post forecasts of cotton production in the study area. The numerical ex-post values of cotton production were used to forecast the trends over the 2006 – 2013 sub-period as retained in the analysis (Table 2). This was based on the linear and grafted models. The result of ex-post forecastof production and supply trends among cotton farmers revealed that the grafted function provided better estimates of cotton production during the sub-period. The values of the grafted (mean) function of 123,000 tons were closer to the observed values of 129,000 tons resulting in smaller difference during the sub-period (2006-2013) under consideration when compared with linear values of 137,000 tons.This is due to the fact that, the grafted (mean) function entrenched the key and major observed local trends in the forecasting framework which has enhanced a more relevant time series prediction over the entire sample period. This type of forecasting may be useful for policy makers and agricultural administrators in planning for future cotton production based on the available information of the past periods.

Table 2: Ex-post Forecasts of Cotton Production in Zamfara State, Nigeria. (2006 – 2013)

Year	Observed Cotton Production ('000 tons)*	Forecasts using the Grafted Function ('000 tons)	Forecast using the Linear Function ('000 tons)
2006	149	165.20	144
2007	215	161.20	142
2008	110	140.40	140
2009	109	128.0	138
2010	111	115.60	136
2011	111	103.20	134
2012	112	90.80	132
2013	115	78.40	130
Total	1032	982.8	1096
Mean	129	123	137

* The observed cotton production trend was obtained from ZADP and other secondary sources.

The result of this work agrees with that of Rahman and Damisa (1999) in forecasting performance of a grafted model on cowpea price in Zaria area of Kaduna State, Nigeria. The result revealed that, the mean trend model predicted better than the linear model because the mean trend model was built on the principle that past local trends in the observed time series (linear, quadratic, exponential etc.) have been incorporated into the body for forecasting immediate future values. This had helped to achieve a more relevant time series prediction that provided appropriate flexible and useful policy guide especially in cases where forecasts are obtained based on estimation.

In the same vein, the result of this analysis is in agreement with the work of Oyakhilomen*et al.* (2013) who estimated the numerical ex-post forecast of rice consumption in Nigeria using the structural parameters of the linear and grafted models to determine their predictive performance with the linear model utilized as a benchmark. The result of their work showed that, the estimates of the grafted model were closer to the observed rice consumption trend in comparison with the estimates of the linear model.

CONCLUSION AND RECOMMENDATIONS

Economic time series data were used in order to estimate the possible production and supply trends. In the application of the grafted polynomial model used to forecast cotton production trends, the estimates of the linear and grafted functions were utilised to obtain *ex-post* forecast of cotton. The values of the grafted (mean) function of 123,000 tons were closer to the observed valueof 129,000 tons resulting in smaller difference during the sub- period (2006-2013) under considerationwhen compared with linear value of 137,000 tons during the same period. Based on these results the following recommendations were made.

- Provision of sufficient incentives by government in terms of inputs and timely inputs delivery system through private inputs voucher distribution network will enhance production.
- Intensive research and extension programme are required to disseminate and increase the level of awareness of farmers on the need to adopt and use improved cotton production technological packages.
- Formation of organized statutory market such as commodity Marketing Corporation and fixing of guaranteed minimum prices by government will encourage farmers to improve on cotton production.

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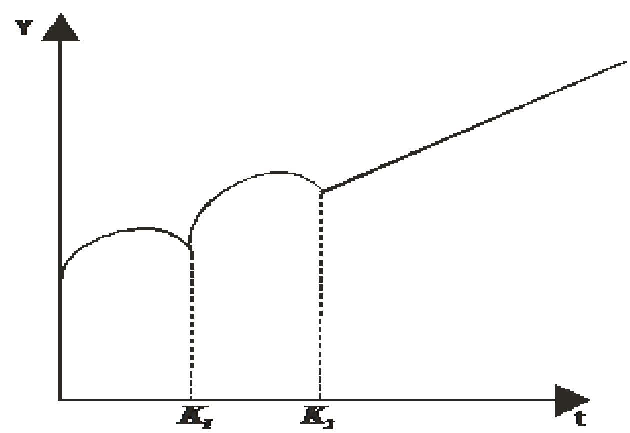
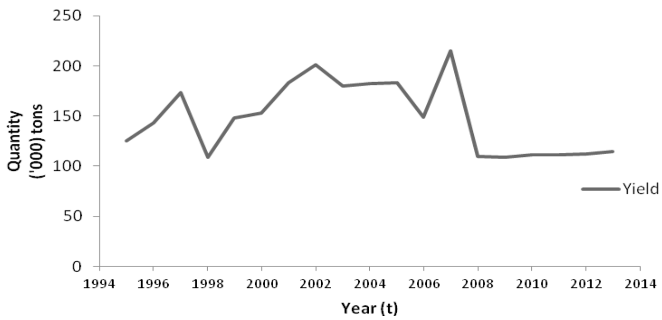
APPENDIX I: QUADRATIC-QUADRATIC-LINEAR POLYNOMIAL FUNCTION

The suggested quadratic-quadratic-linear (Q-Q-L) function is of a graphical form shown below:



APPENDIX 2: COTTON PRODUCTION TREND IN ZAMFARA STATE, NIGERIA (1995 – 2013)



APPENDIX 3: GRAFTED POLYNOMIAL MODEL

This summary presents the derivation of the reduced form of expressions in coefficients a_o, a_1, b_o and b_1 in the grafted polynomial model. These were presented earlier in the study as equations (28) – (31). For ease and simplicity, it is convenient to start our derivations with equation (29) and making b_1 the subject of the formula.

$b_1 + 2b_2k_2 = c_1$

$b_1 = c_1 - 2b_2k_2$ (i)

From equation (30);

$a_1 + 2a_2k_1 = b_1 + 2b_2k_1$

$a_1 = b_1 + 2b_2k_1 - 2a_2k_1$

Substitute for b_1

$a_1 = c_1 - 2b_2k_2 + 2b_2k_1 - 2a_2k_1$

$a_1 = c_1 - 2b_2(k_2 - k_1) - 2a_2k_1$ ii)

From equation (29);

$b_0 + b_1k_2 + b_2k_2^2 = c_0 + c_1k_2$

$b_0 + c_1k_2 - 2b_2k_2^2 + b_2k_2^2 = c_0 + c_1k_2$

$b_0 = c_0 + c_1k_2 - c_1k_2 + 2b_2k_2^2 - b_2k_2^2$

$b_0 = c_0 + b_2k_2^2$

$b_0 = c_0 + b_2k_2^2$ (iii)

*From equation (*28),

$a_0 + a_1k_1 + a_2k_2^2 = b_0 + b_1k_1 + b_2k_1^2$

Substituting for a_1, b_o and b_1

$a_0 + [c_1 - 2b_2k_2 + 2b_2k_1 - 2a_2k_1]\,k_1 + a_2k_1^2 = c_0 + b_2k_2^2 + (c_1 - 2b_2k_2)\,k_1 + b_2k_1^2$

$a_0 + c_1k_1 - 2b_2k_2k_1 + 2b_2k_1^2 - 2a_2k_1^2 + a_2k_1^2 = c_0 + b_2k_2^2 + c_1k_1 - 2b_2k_2k_1 + b_2k_1^2$

$a_0 = c_0 + b_2k_2^2 - b_2k_1^2 + a_2k_1^2$

$= c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2$

$a_0 = c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2$ (iv)

From equation (25)

$= a_0 + a_1t + a_2t^2$

Substituting for a_o and a_1

$= [c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2] + [c_1 - 2b_2(k_2 - k_1) - 2a_2k_1]t + a_2t^2$

$= c_0 + b_2(k_2^2 - k_1^2) + a_2k_1^2 + [c_1 - 2b_2k_2 + 2b_2k_1 - 2a_2k_1]t + a_2t^2$

$= c_0 + b_2k_2^2 - b_2k_1^2 + a_2k_1^2 + c_1t - 2b_2k_2t + 2b_2k_1t - 2a_2k_1t + a_2t^2$

$= c_0 + c_1t + a_2t_1^2 - 2a_2k_1t + a_2k_1^2 + b_2k_2^2 - b_2k_1^2 - 2b_2k_2t + 2b_2k_1t$

$= c_0 + c_1t + a_2(t_1 - 2k_1t + k_2) + b_2(k_2^2 - k_1^2 - 2k_2t + 2k_1t)$

$= c_0 + c_1t + a_2(t - k)^2 + b_2[(k_2^2 - k_1^2) - 2(k_2 - k_1)t$ v)

Equation (v) can be transformed as;

$= c_0x_0 + c_1x_1 + a_2x_2 + b_2x_3$ (vi)

This is amenable and compatible with OLS and can be represented as;

(vii)

Where;

$x_0 = 1;$ for all t,

$x_1 = t;$ for all t

$x_2 = ;$ for $k_1 \leq t$

$= 0;$ otherwise

$x_3 = (k_2^2 - k_1^2) - 2(k_2 - k_1)\,t$ for k_1 t

$= (t - k_2);$ for $k_1 < t$ k_2

$= 0$