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TIME TRUNCATED GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIME PERCENTILES UNDER GENERALIZED LOG-LOGISTIC DISTRIBUTIONS

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ABSTRACT

A group acceptance sampling plan is considered for a truncated life test when a multiple number of items as a group can be tested simultaneously in a tester. Group acceptance sampling plans under the truncated life test are designed for lifetime percentiles when the lifetime of a product follows the generalized log-logistic distribution or the Burr type XII distribution. The design parameters such as the number of groups and the acceptance number required are determined by satisfying the consumer's risk and producer's risk at the specified quality levels, while the number of testers and the termination time are specified. The comparison between the distributions is given using the percentiles life of the products. The results are discussed with real life industrial examples. The extensive tables and graphs are given to explain the procedure developed under the generalized log-logistic distribution or the Burr type XII distribution.

Keywords: Generalized log-logistic distribution, Burr type XII distribution, Percentile, Consumer's risk, producer's risk, truncated life test

1. INTRODUCTION

In practice, it is difficult to test the complete life time of every item from a large lot due to the cost and the time required for the inspection. So, the decision about the acceptance or rejection of submitted lots should be based on sampled items selected from the lot. The single acceptance sampling plan is often adopted in laboratory for the life test purpose due to its simplicity. In this sampling scheme, the capacity to install items on a tester is limited to one. Therefore, the experimenter needs the number of testers equal to the number of items selected from the lot. Obviously, installing a single item to a single tester requires lots of efforts, time, and cost. Saving these resources including cost and time is an important issue in life testing. The cost and the time are the factor which is directly related to the number of items selected from a lot. The larger the sample size the larger the producer's loss. Therefore, researchers have been trying to propose or improve the sampling plan to require smaller sample size in a life testing. Nowadays testers accommodating a multiple number of items at a time are used in practice because testing time and cost can be saved by testing these items simultaneously. For more detail, reader can refer to Aslam and Jun (2009). Items in a tester can be regarded as a group and the number of items in a group is called the group size. The acceptance sampling plan used to determine these groups of items will be called a group acceptance sampling plan (GASP). The sudden death testing scheme is always implemented in groups. Many authors including Pascual and Meeker (1998), Vlcek et al. (2003) and Jun et al. (2006) discussed the sudden death testing in groups. More recently, Aslam and Jun (2009) developed a group acceptance sampling plan based on truncated life test when the lifetime of a product is best fitted to the inverse Rayleigh or log-logistic distribution and Srinivasa Rao (2010) developed a group acceptance sampling plan based on truncated life test for the Marshall-Olkin extended Lomax distribution.

The ordinary acceptance sampling plans and the group acceptance sampling plans based on time truncated life in the literature are proposed using the mean or median life of the product for assuring the quality and reliability of the product. As stated by Lio et al. (2010a) and Lio et al. (2010b), the existing acceptance sapling plans may not assure the engineering consideration on the specific percentile of item life time. When the quality of a major focus is a low percentile, the acceptance sampling plans based on the mean life could not pass a lot which has the low percentile below the required customer standard. Furthermore, a small decrease in the average lifetime with a simultaneous small increase in the variance can result in a significant downward shift in small percentile of interest. This means that a lot of products could be accepted due to a small decrease in the mean life after inspection. The accepted lot may not meet the consumer's expectation if the low percentile is used for the lifetime of products. The mean life may not be applicable to a skewed distribution but the percentile is more suitable to apply on the distribution for making the required results. The median lifetime is suitable when the distribution is skewed. See, for example, Marshall and Olkin (2007) and Aslam et al. (2010).

It is important to note that the items produced under the same environment have some random variation in their lifetimes. This variation in failure time can be modeled by a probability distribution. The life time distribution also plays a vital rule to design an acceptance sampling plan. The ordinary acceptance sampling plans based on the truncated life test using various distributions have been discussed by many authors in the literature including Epstein (1954), Goode and Kao (1961), Kantam and Rosaiah (1998), Kantam et al. (2001), Baklizi (2003), Rosaiah et al. (2006), Rosaiah and Kantam (2005), Tsai and Wu (2006), Rosaiah et al. (2007), Aslam and Kantam (2008), and Balakrishnan et al. (2007).

Two risks are always associated with any type of sampling scheme. The probability of accepting a bad lot is called the consumer's risks and the chance of rejecting a good lot is called the producer's risk. The acceptance sampling schemes including the variable sampling, attribute sampling, skip-lot sampling and the normal to tightened sampling are used to reduce the producer's risk and the consumer's risk. So, the determination of the design parameters such as the sample size and the acceptance number satisfying both risks is preferable to the single-point approach. Further, as stated by Aslam and Jun (2009), a sampling plan obtained by satisfying only the consumer' risk may not always satisfy the producer's risk. The two-point approach on the OC curve for designing the variable acceptance sampling plan has been developed and implemented by Fertig and Mann (1998), Jun et al. (2006).

The main purpose of this paper is to propose a GASP based on truncated life tests when the lifetime of an item follows the generalized log-logistic distribution or the Burr type XII distribution. As the best of author's knowledge, no attention has been paid to use these distributions to develop the group plans for the lifetime percentiles of the product using the two points on OC curve approach.

2. Generalized Log-Logistic and Burr Type XII Distributions

The generalized log-logistic distribution and the Burr type XII distribution are the life time distributions, which have been widely used in the area of reliability and the acceptance sampling plan for the testing purpose. These two distributions are not symmetric and different generalized forms of the log-logistic distribution. The generalized log-logistic distribution was applied to a breast cancer survival data by Singh et al. (1994). The application of the generalized log-logistic distribution in a double acceptance sampling plan is discussed by Aslam and Jun (2010). Kantam et al. (2001) used the log-logistic distribution in acceptance sampling plans. Recently, Lio et al. (2010a) and Lio et al. (2010b) used the Burr type XII distribution and the generalized Birnbaum-Saunders distribution to develop the ordinary acceptance sampling plan using the percentiles as life time. They showed that both distributions are well fitted to real data. To develop the group plan, we will assume that the lifetime of a product either follows the generalized log-logistic distribution or the Burr type XII distribution. The brief introduction of these two distributions is given as follows:

The probability density function (pdf) and the cumulative distribution function (cdf) of the generalized log-logistic distributions are respectively given as

$$f(t) = \frac{b\theta(t/\sigma)^{b\theta-1}}{\sigma\left[1+(t/\sigma)^b\right]^{\theta+1}}, \qquad t \ge 0, \sigma > 0, b > 0, \theta > 0 \tag{1}$$

$$F(t) = \left[\frac{(t/\sigma)^b}{1+(t/\sigma)^b}\right]^{\theta}, \qquad t \ge 0, \sigma > 0, b > 0, \theta > 0 \tag{2}$$

where b is the shape parameter, σ is the scale parameter and θ is the second shape parameter (or index parameter). The cdf given in Eq. (2) represents the defective probability of a parallel system with index parameter θ having a log-logistic distribution. When the index parameter θ=1, the generalized log-logistic distribution converts to the log-logistic distribution. The 100q-th percentile of the generalized log-logistic distribution is as follows:

$$t_q = \sigma\left[\frac{1}{(1/q)^{1/\theta}-1}\right]^{1/b} \tag{3}$$

Particularly, the median life (m) is 50-th percentile, so it is given by

$$m = \sigma\left[\frac{(0.5)^{1/\theta}}{1-(0.5)^{1/\theta}}\right]^{1/b} \tag{4}$$

When the two shape parameters are fixed, the median is proportional to the scale parameter σ. Particularly, the median life of log-logistic distribution is given by regardless of the shape parameter b:

$$m=\sigma \tag{5}$$

The cdf of Burr type XII distribution is given as

$$F(t)=1-\left[1+(t/\eta)^{b}\right]^{-k}, \qquad t\geq 0,\eta>0,b>0,k>0 \tag{6}$$

Here, η is the scale parameter, b and k are the two shape parameters. When k=1, the burr type XII distribution converts to the log-logistic distribution. The 100q-th percentile of burr type XII distribution is given as:

$$t_q=\eta\left[\left(\frac{1}{(1-q)}\right)^{1/k}-1\right]^{1/b} \tag{7}$$

The median life of the Burr type XII distribution is given by:

$$m=\eta\left[\frac{1-(0.5)^{1/k}}{(0.5)^{1/k}}\right]^{1/b} \tag{8}$$

When the two shape parameters are fixed, the median is proportional to the scale parameter η.

3. Proposed Group Sampling Plan

Assume that the 100q-th percentile lifetime, denoted by t_q, is used to represent the quality of the product. We say that the submitted lot has a good quality (and the lot will be accepted) if the null hypothesis, $H_0: t_q \geq t_{q_0}$ (t_{q_0} is a specified life percentile) is accepted against the alternative hypothesis $H_1: t_q < t_{q_0}$. In acceptance sampling schemes, we can test the above stated null hypothesis on the basis of number of failures from the sample. If, for example, the number of failures is larger than c, then H_0 should be rejected in favor of H_1. To test the above stated hypothesis, it is necessary to develop the acceptance sampling plan. We assume that a tester has capacity to install r (≥1) items so that the group size is larger than or equal to one.

We propose the following group acceptance sampling plan based on the truncated life test.

1. Draw the random sample of size n from a lot, allocate r items to each of g groups (or testers) so that $n=rg$ and put them on test for the duration of t_0.

2. Accept the lot if the total number of failures from g groups is smaller than or equal to c. Truncate the test and reject the lot as soon as the total number of failures from g groups exceeds c before t_0.

The above group plan is a generalization of the ordinary plan given in the literature, so it reduces to the ordinary plan if $r=1$. The group plan consists of two major decision parameters which are the number of group g and the acceptance number c. It is important to mention here that r in the above group plan is not a design parameter but it is determined by the type of testers.

The termination time t_0 is assumed to be specified.

The lot acceptance probability is given by

$$L(p) = \sum_{i=0}^{c} \binom{rg}{i} p^i (1-p)^{rg-i}. \tag{9}$$

where *p* is the probability that an item in a group fails before the termination time t_0. It would be convenient to determine the termination time t_0 as a multiple of the specified percentile t_{q_0} such that $t_0 = \delta_q t_{q_0}$ for a constant δ_q. For example $\delta_q = 0.5$ means that the experiment time is just half of the specified life percentile.

The p for the generalized log-logistic distribution based on 100q-th percentile t_q is given by

$$p = \left[\frac{[\delta_q \gamma /(t_q / t_{q_0})]^b}{1 + [\delta_q \gamma /(t_q / t_{q_0})]^b} \right]^{\theta} \tag{10}$$

where

$$\gamma = \left[\frac{1}{(1/q)^{1/\theta} - 1} \right]^{1/b}$$

Particularly, the p based on median life is 50-th percentile is given by

$$p = \left[\frac{(\delta_q \gamma)^b}{(t_q / t_{q_0})^b + (\delta_q \gamma)^b} \right]^{\theta} \tag{11}$$

where $$\gamma = \left[\frac{(0.5)^{1/\theta}}{1 - (0.5)^{1/\theta}} \right]^{1/b} \tag{12}$$

The p based on the 100q-th percentile of the Burr type XII distribution is

$$p = 1 - \left[1 + \left[\frac{\gamma \delta_q}{t_q / t_{q_0}} \right]^b \right]^{-k} \tag{13}$$

where

$$\gamma = \left[(1/(1-q))^{1/k} - 1 \right]^{1/b} \tag{14}$$

Particularly, the p based on the median life is

$$p = 1 - \left[\frac{1}{1 + (\delta_q \gamma /(t_q / t_{q_0}))^b} \right]^k \tag{15}$$

where $$\gamma = \left[\frac{1 - (0.5)^{1/k}}{(0.5)^{1/k}} \right]^{1/b} \tag{16}$$

Therefore, the lot acceptance probability in (9) can be evaluated as we can describe the quality level of a product in terms of the ratio of its 100q-th percentile lifetime to the specified percentile t_q / t_{q_0}.

The probability of rejecting a good lot is called the producer's risk, whereas the probability of accepting a bad lot is known as the consumer's risk. The parameters g and c of the proposed sampling plan are the ones for ensuring the consumer's risk β and producer's risk α. When the quality level based on the ratio mentioned earlier, the two-point approach of finding the design parameters is to determine the minimum number of groups and acceptance number if satisfying these two inequalities

$$L(p \mid t_q / t_{q_0} = r_1) \le \beta \tag{17}$$

$$L(p \mid t_q / t_{q_0} = r_2) \ge 1 - \alpha \tag{18}$$

where, r_1 is the ratio at the consumer's risk and r_2 is the ratio of true percentile to the specified one at the producer's risk. Let p_1 and p_2 are the failure probabilities of corresponding to consumer's and producer's risk. Then, the minimum number of groups and acceptance number can be determined by considering the consumer's risk and producer's risk at the same time through the following inequalities:

$$L(p_2) = \left(\sum_{i=0}^{c} \binom{rg}{i} p_1^{\,i} (1 - p_1)^{rg-i} \right) \le \beta \tag{19}$$

$$L(p_2) = \left(\sum_{i=0}^{c} \binom{rg}{i} p_2^{\,i} (1 - p_2)^{rg-i} \right) \ge 1 - \alpha^* \tag{20}$$

In the below mentioned tables 1-4 show the minimum number of groups and the acceptance number required for the given group acceptance sampling plan according to various values of the consumer's risk $(\beta = 0.25, 0.10, 0.05, 0.01)$ when the true percentile equals the specified life and 5% of producer's risk when the true percentiles are some multiple (ratio) (2,4,6,8) times the specified life. Two level of group size $(r = 5,10)$ and two levels of termination time multiplier $(\delta_q = 0.5, 1.0)$. We consider different values of the β and θ to find the minimum sample size can be obtained, if needed, by $n = r \times g$.

We consider different values of the shape parameters of generalized log-logistic and burr type XII distribution to find the minimum number of groups and acceptance number by using the proposed group sampling plan. The plan parameters are placed in Table 1 and Table 3 for $b = 2, \theta = 3$ and 10th and 50th percentiles, respectively, for the generalized log-logistic distribution. Table 2 and Table 4 show the plan parameters for Burr type XII distribution for $b = 2, \theta = 3$ and 10th and 50th percentiles.

Tables 1-4 are around here

We noted from Tables 1-4 that t_q / t_{q0} increases, the number of groups g reduces for all other same parameters. Similarly, as r increases from 5 to 10, the number of groups reduces.

Example 1 Suppose that the life time of a product is known to follow a generalized log-logistic distribution with $b = 2, \theta = 3$. Suppose that it is desired to develop the group acceptance sampling plan to satisfy that the 50-th percentile lifetime is greater than 1500 h through the experiment to be completed by 1500h using testers taking with five products each. It is assumed that the consumer's risk is 25% when the true 50-th percentile is 1500h and the producer's risk is 5% when the true 50-th percentile is 3000h. Since $\beta = 2, \theta = 3$, the consumer's risk is 0.25, $r = 5, \delta_q = 1.0$ and $t_q / t_{q_0} = 2$, the minimum number of groups and acceptance number can be found as $g = 2$ and $c = 3$ from Table 3. This means that a total of 10 products are needed and that two items will be allocated to each of the five testers. We will accept the lot if no more than three failures occurs before 1500h from two groups.

4. COMPARISON OF DISTRIBUTION

4.1 Comparison of Generalized Log-Logistic and Burr XII Distributions

	Generalized log-logistic			Burr type XII		
t_q / t_{q_0}	c	g	$L(p_2)$	c	g	$L(p_2)$
2	1	69	0.9959	5	36	0.9647
4	0	41	0.9990	1	15	0.9734
6	0	41	0.9999	1	15	0.9943
8	0	41	0.9999	0	9	0.9629
10	0	41	0.9999	0	9	0.9762
12	0	41	0.9999	0	9	0.9834

In the table above, we use the 10^{th} percentile for the comparison purpose between the generalized log-logistic distribution and the burr type XII distribution. The number of groups required related to generalized log-logistic distribution is larger as compared to burr type XII distribution when $\beta = 0.10$ and $r = 10, \delta_q = 0.5$ The probability of acceptance increases in generalized log-logistic distribution as compared to burr type XII distribution. The acceptance number for the generalized log-logistic distribution is smaller as compared to burr type XII distribution by using the 50-th percentile.

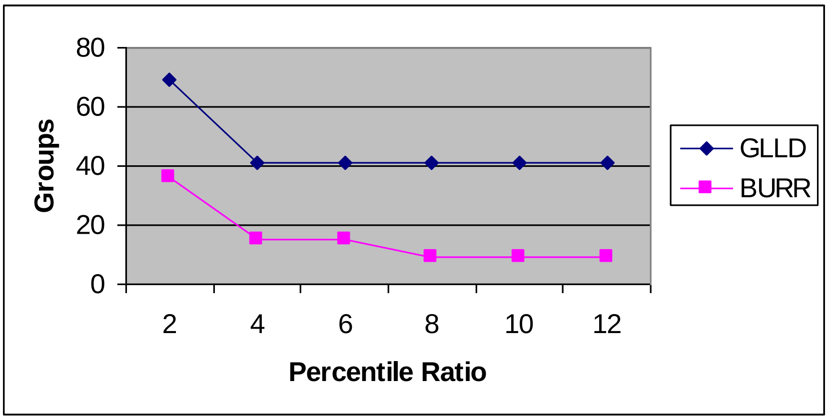

Figure1. Percentile ratio vs. number of groups for two distributions when β =0.10

Figure1 shows that the number of groups as a function of the ratio t_q / t_{q_0} when β =0.10 and δ_q =0.5 for the generalized log-logistic distribution and the burr type XII distribution. It is seen that the number of groups from generalized log-logistic distribution is larger as compared to burr type XII distribution.

4.2 Comparison of Log-Logistic and Weibull Distributions Using 50-th percentile for β =0.10, r =5 and a =0.5

	log-logistic			Weibull		
t_q / t_{q_0}	c	g	$L(p_2)$	c	g	$L(p_2)$
2	5	9	0.9528	5	12	0.9587
4	1	4	0.9626	1	5	0.9705
6	1	4	0.9917	1	5	0.9936
8	1	4	0.9973	0	3	0.9602
10	0	3	0.9632	0	3	0.9743
12	0	3	0.9743	0	3	0.9821

In the table above, we compare the plan parameters for the log-logistic distribution and the Weibull distribution when $\beta = 0.10$ and $r = 5, \delta_q = 0.5$. The probability of acceptance increases a little in log-logistic distribution as compared to the Weibull distribution The acceptance number for the log-logistic distribution is smaller as compared to Weibull distribution by using the 50th percentile. The median of Weibull distribution are given as $m = \lambda \left[\ln(2)\right]^{1/k}$.

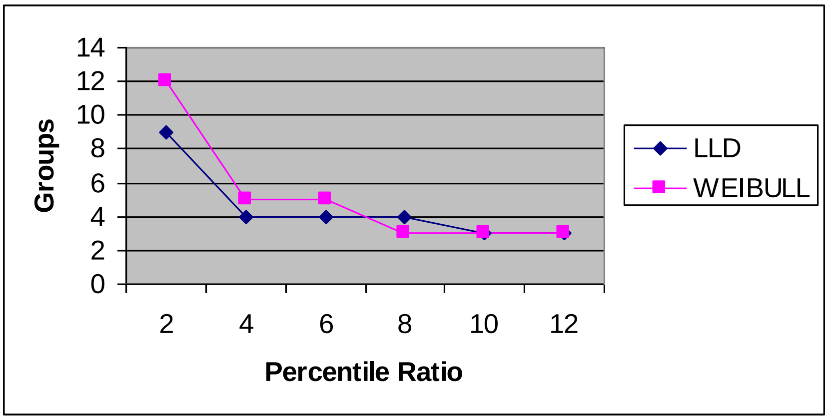

Figure 2. Percentile ratio vs. Groups for log-logistic distribution and the Weibull distribution $\beta = (0.10)$.

Figure 2 shows that the number of groups as a function of the ratio t_q / t_{q_0} when β =0.10 and δ_q =0.5, $r = 5$ for the log-logistic distribution and the Weibull distribution. It is seen that the number of groups from log-logistic distribution is minimum as compared to Weibull distribution.

5. CONCLUSION

We develop the group acceptance sampling plan based on a truncated life test under the assumption that the lifetime of a product follows the generalized log-logistic distribution and burr type XII distribution with known and unknown shape parameter. The two point approach was used for determining the design parameters such as the number of groups and the acceptance number. Our proposed plan indicate that the generalized log-logistic distribution provide the larger number of groups as compared to burr type XII distribution by using the 10^{th} percentile but in 50^{th} percentile the two distribution groups are not quite different. The log-logistic distribution is better than the Weibull distribution. As in acceptance sampling schemes, there is still a capacity available to reduce sample size to save the time and the cost of the experiment. Therefore, there is need to modify the proposed plan using the percentiles of the distributions as a future research.

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Table 1: Proposed group sampling plans for the generalized log-logistic distribution using percentile $q_{0.1}$ and $\beta=2, \theta=3$

β	t_q/t_{q_0}	r=5						r=10					
		δ_q =0.5			δ_q =1.0			δ_q =0.5			δ_q =1.0		
		g	c	L(P2)	g	c	L(P2)	g	c	L(P2)	g	c	L(P2)
0.25	2	50	0	0.9668	6	1	0.9875	25	0	0.9668	3	1	0.9875
	4	50	0	0.9994	3	0	0.9979	25	0	0.9994	2	0	0.9973
	6	50	0	0.9999	3	0	0.9998	25	0	0.9999	2	0	0.9997
	8	50	0	0.9999	3	0	0.9999	25	0	0.9999	2	0	0.9999
0.10	2	138	1	0.9959	8	1	0.9785	69	1	0.9959	4	1	0.9785
	4	82	0	0.9990	5	0	0.9966	41	0	0.9990	3	0	0.9959
	6	82	0	0.9999	5	0	0.9997	41	0	0.9999	3	0	0.9996
	8	82	0	0.9999	5	0	0.9999	41	0	0.9999	3	0	0.9999
0.05	2	168	1	0.9940	10	1	0.9674	84	1	0.9940	5	1	0.9674
	4	106	0	0.9987	6	0	0.9959	53	0	0.9987	3	0	0.9959
	6	106	0	0.9998	6	0	0.9996	53	0	0.9998	3	0	0.9996
	8	106	0	0.9999	6	0	0.9999	53	0	0.9999	3	0	0.9999
0.01	2	235	1	0.9887	17	2	0.9874	118	1	0.9886	9	2	0.9854
	4	163	0	0.9981	9	0	0.9939	82	0	0.9980	5	0	0.9933
	6	163	0	0.9998	9	0	0.9994	82	0	0.9998	5	0	0.9994
	8	163	0	0.9999	9	0	0.9999	82	0	0.9999	5	0	0.9999

Table 2: Proposed group sampling plans for the Burr type XII distribution using percentile $q_{0.1}$ and $\beta=2, \theta=3$

β	t_q/t_{q_0}	r=5						r=10					
		δ_q =0.5			δ_q =1.0			δ_q =0.5			δ_q =1.0		
		g	c	L(P2)	g	c	L(P2)	g	c	L(P2)	g	c	L(P2)
0.25	2	39	3	0.9574	13	4	0.9715	20	3	0.9539	7	4	0.9625
	4	21	1	0.9864	6	1	0.9829	11	1	0.9851	3	1	0.9829
	6	11	0	0.9599	3	0	0.9563	6	0	0.9563	3	1	0.9964
	8	11	0	0.9772	3	0	0.9752	6	0	0.9752	2	0	0.9670
	2	71	5	0.9667	19	5	0.9601	36	5	0.9647	10	5	0.9508

0.10	4	30	1	0.9734	8	1	0.9706	15	1	0.9734	4	1	0.9706
	6	30	1	0.9943	8	1	0.9936	15	1	0.9943	4	1	0.9936
	8	18	0	0.9629	5	0	0.9589	9	0	0.9629	3	0	0.9509
0.05	2	-	-	-	-	-	-	45	6	0.9668	12	6	0.9599
	4	36	1	0.9629	10	1	0.9588	18	1	0.9629	5	1	0.9558
	6	36	1	0.9918	10	1	0.9902	18	1	0.9918	5	1	0.9902
	8	23	0	0.9529	6	0	0.9509	12	0	0.9509	3	0	0.9509
0.01	2	-	-	-	-	-	-	66	8	0.9646	17	8	0.9629
	4	64	2	0.9828	17	2	0.9805	32	2	0.9828	9	2	0.9773
	6	50	1	0.9847	13	1	0.9837	25	1	0.9847	7	1	0.9813
	8	50	1	0.9949	13	1	0.9946	25	1	0.9949	7	1	0.9937

Note: The cells with hyphens (-) indicate that g and c are found to be large.

Table 3: Proposed group sampling plans for the generalized log-logistic distribution when $\beta = 2,\ \theta = 3$ using 50-th percentile.

β	t_q / t_{q_0}	r=5						r=10					
		δ_q =0.5			δ_q =1.0			δ_q =0.5			δ_q =1.0		
		g	c	L(P2)	g	c	L(P2)	g	c	L(P2)	g	c	L(P2)
0.25	2	5	1	0.9857	2	3	0.9775	3	1	0.9798	1	3	0.9775
	4	3	0	0.9973	1	0	0.9641	2	0	0.9964	1	1	0.9977
	6	3	0	0.9997	1	0	0.9955	2	0	0.9996	1	0	0.9910
	8	3	0	0.9999	1	0	0.9991	2	0	0.9999	1	0	0.9982
0.10	2	7	1	0.9731	3	4	0.9753	4	1	0.9655	2	5	0.9760
	4	4	0	0.9964	1	0	0.9641	2	0	0.9964	1	1	0.9977
	6	4	0	0.9996	1	0	0.9955	2	0	0.9996	1	0	0.9910
	8	4	0	0.9999	1	0	0.9991	2	0	0.9999	1	0	0.9982
0.05	2	8	1	0.9655	4	5	0.9760	4	1	0.9655	2	5	0.9760
	4	5	0	0.9955	1	0	0.9641	3	0	0.9946	1	1	0.9977
	6	5	0	0.9996	1	0	0.9955	3	0	0.9995	1	0	0.9910
	8	5	0	0.9999	1	0	0.9991	3	0	0.9999	1	0	0.9982
0.01	2	14	2	0.9853	-	-	-	7	2	0.9853	3	8	0.9938
	4	8	0	0.9927	3	1	0.9948	4	0	0.9927	2	1	0.9908
	6	8	0	0.9993	2	0	0.9910	4	0	0.9993	1	0	0.9910
	8	8	0	0.9998	2	0	0.9982	4	0	0.9998	1	0	0.9982

Note: The cells with hyphens (-) indicate that g and c are found to be large.

Table 4: Proposed group sampling plans for the Burr type XII distribution when $\beta = 2, \theta = 3$ using 50-th percentile.

β	t_q / t_{q_0}	r=5						r=10					
		δ_q =0.5			δ_q =1.0			δ_q =0.5			δ_q =1.0		
		g	c	L(P2)	g	c	L(P2)	g	c	L(P2)	g	c	L(P2)
0.25	2	8	4	0.9609	3	5	0.9683	4	4	0.9609	2	6	0.9566
	4	3	1	0.9862	1	1	0.9797	2	1	0.9759	1	2	0.9902
	6	3	1	0.9971	1	1	0.9956	2	1	0.9948	1	1	0.9817
	8	2	0	0.9700	1	1	0.9986	1	0	0.9700	1	1	0.9938
0.10	2	11	5	0.9555	-	-	-	6	6	0.9774	2	6	0.9566
	4	5	1	0.9635	2	2	0.9902	3	2	0.9944	1	2	0.9902
	6	5	1	0.9919	2	1	0.9817	3	1	0.9885	1	1	0.9817
	8	3	0	0.9554	2	1	0.9938	3	1	0.9962	1	1	0.9938
0.05	2	-	-	-	-	-	-	7	6	0.9531	3	10	0.9914
	4	7	2	0.9913	3	2	0.9688	4	2	0.9875	2	3	0.9869
	6	6	1	0.9885	2	1	0.9817	3	1	0.9885	1	1	0.9817
	8	6	1	0.9962	2	1	0.9938	3	1	0.9962	1	1	0.9938
0.01	2	-	-	-	-	-	-	10	8	0.9529	-	-	-
	4	10	2	0.9773	3	2	0.9688	5	2	0.9773	2	3	0.9869
	6	10	2	0.9975	3	1	0.9602	4	1	0.9802	2	2	0.9915
	8	8	1	0.9933	3	1	0.9862	4	1	0.9933	2	1	0.9759

Note: The cells with hyphens (-) indicate that g and c are found to be large.