Review Article

Significance of Pre A*-Subalgebra

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ABSTRACT

Pre A*-subalgebra is defined and intersection of Pre A*-subalgebras of a Pre A*-algebra A is a Pre A*-subalgebra, every element of Pre A*-subalgebra generated by X of A has a DNF have been proved., Standard element, Standard Ideals and Standard Pre A*-subalgebra are defined and proved that Standard Pre A*-subalgebra have similar characterization to those of Standard ideals

Key Words: Pre A*-subalgebra, DNF, Standard Ideals and Standard Pre A*-subalgebra

INTRODUCTION

In 1994, P. Koteswara Rao[4] first introduced the concept of A*-algebra $\left(A,\wedge,\vee,*,(-)^{\sim}(-)_{\pi},0,1,2\right)$. In 2000, J. Venkateswara Rao[5] introduced the concept Pre A*-algebra $\left(A,\wedge,\vee,(-)^{\sim}\right)$ analogous to C-algebra as a reduct of A*- algebra, studied their sub direct representations, obtained the results that **2**={0, 1} and **3**={0, 1, 2} are the sub directly irreducible Pre-A*-Algebras.

Boolean algebra depends on two-element logic. C-algebra, Ada, A*- algebra and our Pre A*-algebra are regular extensions of Boolean logic to three truth-values, where the third truth-value stands for an undefined truth-value. In [7] partial ordering on a Pre A* -algebra A has been derived with necessary conditions to become a lattice. In [7] congruence relation on Pre A*-Algebra and studied the properties, in [7] ternary operation on Pre-A* algebra have been proved and studied the properties and established Cayley's Theorem on Centre of a Pre A*-Algebra, studied [7] Decomposition of Pre A*-algebra.

In [6] definitions of ideal, prime ideal, maximal ideal of Pre A*-algebra are formulated and studied the properties.

In [8] Pre A*-algebra function is defined along with the table of partial list. In this paper Pre A*-subalgebra is defined and intersection of any family of Pre A*-subalgebra of a Pre A*-algebra A is a Pre A*-subalgebra, every element of Pre A*-subalgebra generated by X of A has a DNF have been proved. Standard Ideals and Standard Pre A*-Subalgebra are defined and proved that Standard Pre A*-Subalgebra have similar Characterization to those of Standard Ideals.

1. PRELIMINARIES

Definition 1.1:

An algebra $(A,\wedge,\vee,(-)^{\sim})$ where A is non-empty set with 1, $\wedge$, $\vee$ are binary operations and $(-)^{\sim}$ is a unary operation satisfying

(a) $x^{\sim\sim}=x, \quad \forall x\in A$

(b) $x\wedge y=y\wedge x, \quad \forall x,y\in A$

(c) $x\wedge y=y\wedge x, \quad \forall x,y\in A$

(d) $(x\wedge y)^{\sim}=x^{\sim}\vee y^{\sim}, \quad \forall x,y\in A$

(e) $x\wedge(y\wedge z)=(x\wedge y)\wedge z, \quad \forall x,y,z\in A$

(f) $x\wedge(y\vee z)=(x\wedge y)\vee(x\wedge z), \quad \forall x,y,z\in A$

(g) $x\wedge y=x\wedge(x^{\sim}\vee y), \quad \forall x,y\in A$.

is called a Pre A*-algebra.

Example 1.2:

3 = {0, 1, 2} with operations $\wedge,\vee,(-)^{\sim}$ defined below is a Pre A*-algebra. This is shown in table-1

Note 1.3.: The elements 0, 1, 2 in the above example satisfy the following laws:

(a) $2^{\sim}=2$ (b) $1\wedge x=x$ for all $x\in 3$

(c) $0\vee x=x$, $\forall\ x\in \mathbf{3}$ (d) $2\wedge x=2\vee x=2$, $\forall\ x\in\mathbf{3}$.

Example 1.4: 2= {0, 1} with operations $\wedge,\vee,(-)^{\sim}$ defined below is a Pre A*-algebra.

This is shown in table-2

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Note 1.5:

(i) $(\mathbf{2}, \wedge, \vee, (-)^{\sim})$ is a Boolean algebra. So every Boolean algebra is a Pre A* algebra
(ii) The identities 1.1(a) and 1.1(d) imply that the varieties of Pre A*-algebras satisfies all the dual statements of 1.1

Lemma 1.6: Every Pre A*-algebra with 1 satisfies the following laws

(a) $x \vee 1 = x \vee x^{\sim}$
(b) $x \wedge 0 = x \wedge x^{\sim}$

Lemma 1.7: Every Pre A*-algebra with 1 satisfies the following laws.

(a) $x \wedge (x^{\sim} \vee x) = x \vee (x^{\sim} \wedge x) = x$
(b) $(x \vee x^{\sim}) \wedge \mathrm{y} = (\mathrm{x} \wedge \mathrm{y}) \vee (\mathrm{x}^{\sim} \wedge \mathrm{y})$
(c) $(x \vee y) \wedge z = (x \wedge z) \vee (x^{\sim} \wedge \mathrm{y} \wedge \mathrm{z})$

Definition 1.8: Let A be a Pre A*-algebra. An element $x \in A$ is called central element of A if $x \vee x^{\sim}=1$ and the set $\{ x \in A / x \vee x^{\sim}=1 \}$ of all central elements of A is called the centre of A and it is denoted by $B(A)$.

Note that if A is a Pre A*- algebra with 1, then $1,0 \in B(A)$. If the centre of a Pre A*- algebra coincides with $\{0,1\}$ then we say that A has trivial centre.

Theorem 1.9: Let $B(A)$ be a Pre A*-algebra with 1, then $B(A)$ is a Boolean algebra with the induced operations $\wedge, \vee, (\text{-})^{\sim}$

Theorem 1.10: Let A is a Pre A*-algebra with 1. Then A has trivial centre if and only if $A = \overline{A}_o$, for some Pre A*-algebra A_o.

Lemma 1.11: Let A be a Pre A*-algebra with 1 ,

(a) If $y \in B(A)$ then $x \wedge x^{\sim} \wedge \mathrm{y} = x \wedge x^{\sim}, \forall x \in \mathrm{A}$
(b) If $x, y \in B(\mathrm{A})$ then $x \wedge (x \vee y) = x \vee (x \wedge y) = x$

Lemma 1.12: Let A be a Pre A*algebra with 1, 0 and let $x \vee y = 0$,

(a) If $x \vee y = 0$, then $x = y = 0$
(b) If $x \vee y = 1$, then $x \vee x^{\sim} = 1$

Theorem 1.13: Let A be a Pre A*-algebra with 1 and $x \wedge y = 0$, if $x \wedge y = 0$, $x \vee y = 1$, then $y = x^{\sim}$

Definition1.14: A nonempty subset U of a Pre A*- algebra A is said to be a dual ideal (filter) of A if the following hold

$a, b \in U \Rightarrow a \wedge b \in U$

$a \in U \Rightarrow x \vee a \in U$ for each x∈A

The two conditions are clearly dual of those in the definition of an ideal of Pre A* - algebra.

Definition 1.15: A Pre A*-algebra function is said to be in disjunctive normal form in n variables.

$x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n$ if it can be written as join of terms of the type $f_1(x_1) \wedge f_2(x_2) \wedge \ldots\ldots\ldots f_n(x_n)$ where $f_i(x_i) = x_i \; or \; x_i^{\sim} \quad \forall i = 1 \; to \; n$ and no two terms are same.

$f_1(x_1) \vee f_2(x_2) \vee \ldots\ldots\ldots f_n(x_n)$ are called minterms or minimal polynomials.

Thus a minterm in n variables is a product of n literals in which each variable is represented by the variable itself or its complement.

Definition1.16 : If a DNF contains all the possible minterms then it is complete DNF.

Definition 1.17: A Pre A*-algebra function is said to be in conjunctive normal form in n variables. $f_1(x_1) \vee f_2(x_2) \vee \ldots\ldots\ldots f_n(x_n)$ where $f_i(x_i) = x_i \; or \; x_i^{\sim} \quad \forall i = 1 \; to \; n$ and no two terms are same $f_1(x_1) \vee f_2(x_2) \vee \ldots\ldots\ldots f_n(x_n)$ are called maxterms or maximal polynomials.

2. PRE A*-SUBALGEBRA

Definition 2.1 Let A be a Pre A*-algebra and S is a non-empty subset of A. We define S as a Pre A*-subalgebra if it satisfies the following conditions

If $a \in S$ then $a^{\sim} \in S$

If $a \vee b \in S$ then $a \vee b \in S$

If $a, b \in S$ then $a \wedge b \in S$

Note 2.2: conditions 1 and 2 imply conditions 1 and 3 and vice versa.

By the conditions, $x \vee 1 = 1;\; x \wedge 2 = 2;\; x \vee 2 = 2$ $x \vee 1 = 1;\; x \wedge 2 = 2;\; x \vee 2 = 2$ for any $x \in A$,the four elements $x \wedge x^{\sim}, x, x^{\sim}, x \vee x^{\sim}$ constitute Pre A*-subalgebra of A. This Pre A*-subalgebra is generated by $x^{\sim}$ (or $x^{\sim}$).

Theorem 2.3: Any Pre A*-subalgebra S of a Pre A*-algebra is itself a Pre A*-algebra under the operation of A.

Proof: Each of the postulates of Pre A*-algebra holds in S and its terms are all in $S \subseteq A$, Since S is a Pre A*-algebra with respect to operations defined in the Pre A*-algebra.

Theorem 2.4:Arbitrary intersections of Pre A*-subalgebra i.e., the intersection of any family of Pre A*-subalgebra of a Pre A*-algebra A is a Pre A*-subalgebra.

Proof: Let X be an arbitrary subset of A. Then to prove that the intersection of all Pre A*-subalgebras of A containing X is a Pre A*-algebra. Let $\{Y_i : i \in T\}$ be any family of Pre A*-subalgebra of A. Here T is an index set and is such that $\forall i \in T$, Y_i is a Pre A*-subalgebra of A.

Let $< X >= \{ \bigcap_{i \in T} Y_i / X \subseteq Y_i$ & Y_i is a Pre A*-subalgebra of A $\forall i \in T\}$

be the intersection of family of Pre A*-subalgebras of A. Then to prove that < X> is also a Pre A*-subalgebra of A.

Obviously < X> is a non empty subset of A.

Now let *a,b* be any two elements of <X>.

$$a \in < X > \Rightarrow a \in \bigcap Y_i \Rightarrow a \in Y_i$$

$$b \in < X > \Rightarrow b \in \bigcap Y_i \Rightarrow b \in Y_i$$

Since Y_i is a Pre A*-subalgebra of A,

$$a \in Y_i, b \in Y_i \Rightarrow a \wedge b^{\sim} \in Y_i$$

consequently $a \wedge b^{\sim} \in \bigcap_{i \in T} Y_i$

Thus we have shown that

$$a \in Y_i, b \in Y_i \Rightarrow a \wedge b^{\sim} \in \bigcap Y_i$$

Therefore $\bigcap Y_i$ is a Pre A*-subalgebra of A.

This shows that < X > is a Pre A*-subalgebra of A.

Note 2.5:

- The intersection of all Pre A*-subalgebras containing X is called Pre A*-subalgebra generated by X and is denoted by < X >.

Since X is a subset of each member of the set whose intersection is < X >, $X \subseteq < X >$. < X > is a Pre A*-subalgebras containing X. Thus < X > is the least Pre A*-subalgebras containing X.

The union of two Pre A*-subalgebras is not necessarily a Pre A*-subalgebra.

If < X > = A, then Pre A*-algebra A itself is generated by X.

Theorem 2.6 : Let A be a Pre A*-algebra. Every element of Pre A*-subalgebra generated by X of A has a DNF.

Proof: If $X \in < X >$ then $x = \vee_{i=1}^{n} [\wedge_{j=1}^{m} x_{ij}] \quad \forall x \in X$

That is

$$x = (x_{11} \wedge x_{12} \wedge ..... \wedge x_{1m}) \vee (x_{21} \wedge x_{22} \wedge ..... \wedge x_{2m}) \vee ....$$

$$\vee (x_{n1} \wedge x_{n2} \wedge ..... \wedge x_{nm})$$

Let E be the set of all elements written in DNF using elements of X. Clearly $E \subseteq < X >$

Then we have to show that $< X > \subseteq E$. Note that join of two elements in E is in E and the complement of an element in E is also in E. We prove this in two steps.

If $x \in E$ and either y or $y^{\sim}$ is in X then $x \wedge y \in E$

If $x = (x_{11} \wedge x_{12} \wedge ..... \wedge x_{1m}) \vee (x_{21} \wedge x_{22} \wedge .....$

$$y \wedge x = y \wedge (x_{11} \wedge x_{12} \wedge ..... \wedge x_{1m})$$

$$y \wedge x = y \wedge (x_{11} \wedge x_{12} \wedge ..... \wedge x_{1m})$$

$$......... \vee (x_{n1} \wedge x_{n2} \wedge ..... \wedge x_{nm})$$

$$......... \vee (x_{n1} \wedge x_{n2} \wedge ..... \wedge x_{nm})$$

$$= \{y \wedge (x_{11} \wedge x_{12} \wedge ..... \wedge x_{1m})\} \vee$$

$$\{y \wedge (x_{21} \wedge x_{22} \wedge ..... \wedge x_{2m})\} \vee .....$$

$$... \vee \{y \wedge (x_{n1} \wedge x_{n2} \wedge ..... \wedge x_{nm})\}$$

which is in DNF using elements of X.

Therefore if $a,b \in E$ then $a \wedge b \in E$

If $x \in E$ then $x^{\sim} \in E$

If $x = (x_{11} \wedge x_{12} \wedge ..... \wedge x_{1m}) \vee (x_{21} \wedge x_{22}$

$$\vee (x_{n1} \wedge x_{n2} \wedge ..... \wedge x_{nm})$$

$$\vee (x_{n1} \wedge x_{n2} \wedge ..... \wedge x_{nm})$$

$$x^{\sim} = (x_{11}^{\sim} \vee x_{12}^{\sim} \vee .... \vee x_{1m}^{\sim}) \wedge$$

$$(x_{21}^{\sim} \vee x_{22}^{\sim} \vee ..... \vee x_{2m}^{\sim}) \wedge ....$$

$$\wedge (x_{n1}^{\sim} \vee x_{n2}^{\sim} \vee ..... \vee x_{nm}^{\sim})$$

which is the meet of n elements in E.

Therefore we can write $x \in E$

If $x \in E$, $x^{\sim} \in E$ then $\therefore < X > = E$

Since E is closed under join and complementation E is a Pre A*-subalgebra of A.

Therefore E contains X, $\therefore < X > = E$

$\therefore < X > = E$

3. STANDARD IDEAL

Definition 3.1: An element a of a Pre A*-algebra A is called standard element if $b \wedge (a \vee c) = (b \wedge a) \vee (b \wedge c)$ holds for any pair of elements b and c of A.

Definition 3.2: An ideal I of a Pre A*-algebra A is called standard ideal if $H \wedge (I \vee J) = (H \wedge I) \vee (H \wedge J)$ holds for any pair of ideals H and J of A.

Remark 3.3: If I and J are both ideals (or both dual ideals) then $I \wedge J$ and $I \wedge J$ are exactly the join and meet of I and J in the Pre A*-algebra.

Definition 3.4: A Pre A*-subalgebra S of a Pre A*-algebra A is called standard Pre A*-subalgebra if

$$H \wedge (S, J) = (H \wedge S, H \wedge J) \tag{1}$$

$$H \vee (S, J) = (H \vee S, H \vee J) \quad (2)$$

holds for any pair of ideals of Pre A*-subalgebra of A whenever neither $S \wedge J$ nor $H \wedge (S, J)$ are empty.

Proposition 3.5: For each S in Pre A*-subalgebra of A , S is a standard Pre A*-subalgebra of A

Proof: Let H and J are pair of ideals of A.

$S \wedge J \neq \phi \Rightarrow S \in J$ yielding $S \subseteq J$ and

$H \wedge S \subseteq H \wedge J$

$H \vee S \subseteq H \vee J$

Thus in (1) and (2) both left and right hand sides of the equations are $H \vee J$ and $H \vee J$ respectively.

Proposition 3.6: An ideal S of a Pre A*-algebra A is standard if and only if it is a standard Pre A*- subalgebra.

Proof: Let us assume , first that an ideal S is a standard Pre A*-subalgebra of A. Then the ideals H and J are , of course Pre A*- subalgebras.

Moreover $S \wedge J \neq \phi$ and $H \wedge < S, J > \neq \phi$ are clearly satisfied. Thus

$H \vee (S, J) = (H \vee S, H \vee J)$

$H \vee (S, J) = (H \vee S, H \vee J)$

Since we know that $(I, J) = I \vee J$ for the ideals of I and J.

Therefore $H \vee (S \wedge J) = (H \vee S) \wedge (H \vee J)$

$H \vee (S \wedge J) = (H \vee S) \wedge (H \vee J)$

The first equality gives us that S is a standard ideal

Conversely S be a standard ideal.

$H \wedge (S \vee J) = (H \wedge S) \vee (H \wedge J)$

H and J are any pair of ideals of A.

Let $S \wedge J \neq \phi$

that is $S \subseteq J \Rightarrow S \subseteq (S, J) \ \therefore (S, J) = J$

$H \wedge S \subseteq H \wedge J$ and $H \wedge S \subseteq H \wedge J$

$S \subseteq (S, J) \Rightarrow H \vee S \subseteq H \vee (S, J)$

$\therefore (H \vee S, H \vee J) = H \vee J$

$\therefore (H \vee S, H \vee J) = H \vee J$

$H \vee (S, J) = H \vee J$

$\therefore (H \vee S, H \vee J) = H \vee (S, J)$

Similarly

$H \wedge S \subseteq H \wedge (S, J)$

$\Rightarrow H \wedge (S, J) \neq \phi$

Therefore S is a standard Pre A*-subalgebra.

Thus we proved that Standard Pre A*-subalgebra have similar characterizations to those of standard ideals.

CONCLUSION

We defined Pre A*-subalgebra and intersection of Pre A*-subalgebras of a Pre A*-algebra A is a Pre A*-subalgebra. Every element of Pre A*-subalgebra generated by X of A has a DNF has been proved., Standard element, Standard Ideals and Standard Pre A*-subalgebra are defined and proved that Standard Pre A*-subalgebra have similar characterization to those of Standard ideals.

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Table 1: Operations of Boolean algebra

$\wedge$	0	1
0	0	0
1	0	1

$\vee$	0	1
0	0	1
1	1	1

x	$x^{\sim}$
0	1
1	0

Table 2: Operations of Pre A*- algebra

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^{\sim}$
0	1
1	0
2	2