Review Article

IJCRR
Section: General Science
Sci. Journal Impact Factor 4.016
ICV: 71.54

Power Sums Through Mathematical Induction

R. Malathi[1], C. Tamizharasi[1]

[1]Department of Mathematics, SCSVMV University.

ABSTRACT

We observe that Faulhaber's theorem on sums of odd powers holds an random arithmetic progression, the odd power sums of any arithmetic progression: a+b, a+2b,…, a+nb is a polynomial in *n* a + n(n+1)b/2. This assertion can be presumed from the original Faulhaber's theorem. We use the Bernoulli polynomials for the alternative formula. By using the central factorial numbers as in the approach of knuth, we can derive the formulas for r – fold sums of powers without restoring the notion of r – reflective functions. We can also provide formulas for the r-fold alternating sums of powers in terms of Euler polynomials. In its simplest case, a power sum is a sum of the form $S_n(l) = 1^n + 2^n + \dots + (l-1)^n$

Their sums have interesting combination of a number theoretical importance, and were already known by Bernoulli's, but we shall see that even today there are many things about them to discover.

- Basics of power sums
- Generalizations of power sums
- Alternating power sums

In this paper, we are going to discuss about the negative power sum.

Key Words: Infinite sum, Partial sums, Euler polynomial

INTRODUCTION

A very simple application of Cauchy's condition permits us to deduce the convergence of one sequence from that of another. If it is true that $|b_m-b_n|$ $|a_m-b_n|$ for all pairs of subscripts, the sequence $\{a_n$ (This is not a standard term).

Under this condition, if $\{a_n\}$is a Cauchy's sequence, so is $\{b_n\}$.

An infinite series is a formal infinite sum.

$$a_{1+} a_2 + \dots + a_n + \dots \qquad (1)$$

Associated with this series is the sequence of its partial sums.

$$S_n = a_1+a_2+\dots+a_n$$

The series is said to converge iff the corresponding sequence is convergent and if this is the case the limit of the sequence is the sum of the series.

Applied to a series Cauchy's convergence test yields the following condition:

- The series converges iff to every > 0 there exists an n_0 such that $|a_n+a_{n+1}+\dots+a_{n+p}| < \in$ for all $n \geq n_0$ and $p \geq 0$ for $p = 0$ we find in particular that $|a_n| < \in$.

Hence the general term of a convergent series tends to 0. This condition is necessary but of course not sufficient.

If a finite number of the terms of the series 1+2+3+…+n are omitted, the new series converges or diverges together with the series. In the case of convergence, let R_n be the sum of the series which begins with the term a_{n+1}. Then the sum of the whole series is $S=S_n+R_n$.[2]

The series 1+2+3+…+n can be compared with the series,

$$|a_1|+|a_2|+\dots+|a_n|+\dots \qquad (2)$$

Formed by the absolute values of the terms. The sequence of partial sums of (1) is a contraction of the sequence equivalent

Corresponding Author:
R. Malathi, Department of Mathematics, SCSVMV University; E-mail: malathihema@yahoo.co.in

ISSN: 2231-2196 (Print) **ISSN:** 0975-5241 (Online)
Received: 18.02.2017 **Revised:** 04.04.2017 **Accepted:** 25.04.2017

to (2), form $|a_n|+|a_{n+1}|+\ldots+|a_{n+p}|$.

Therefore, convergence of (2) implies that the original series (1) is convergent. A series with the property that the series formed by the absolute values of the terms converges is said to be absolutely convergent.[4]

Some Finite series n

$1+2+3+\ldots+n=$

$n(2k+n-1$

$2+4+6+\ldots+2n = n(n+1)$

$1+3+5+\ldots+(2n-1) = n^2$

$k+(k+1)+(k+2)+\ldots+(k+n-1) = \frac{n(2k+n-1)}{2}$

$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(n+2)}{6}$

$1^2+3^2+5^2+\ldots+(2n-1)^2=n(4n^2-1)/3$

$1^3+3^3+5^3+\ldots+(2n-1)^3=n^2(2n^2-1)$

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}+\ldots= 2$

$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}+\ldots=1$

$1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{(n-1)!}+\ldots=e$

Prove by Mathematical Induction

$(-1)+(-2)+(-3)+\ldots+(-n)=-[n(n+1)/2]$ Put $n= 1$

P(1) = -1 is the statement: $-1= -[1(1+1)/2]$

$-1= -1$

$\therefore$ P(1) is true.

Assume n = k is true for that statement.

(i.e,) assume P(k) is true.

(i.e,) assume $(-1)+(-2)+(-3)+\ldots(-k)$

$= -[k(k+1)/2] \rightarrow(1)$ be true

To prove P(k+1)is true.

(i.e,) to prove $(-1)+(-2)+(-3)+\ldots+(-k)+$

$(-(k+1)) = -[(k+1)(k+2)/2]$ is true.

$[(-1)+(-2)+(-3)+\ldots+(-k)]+(-(k+1))=$

$-[(k(k+1)/2)+(k+1)]$ from(1)

$= -[(k(k+1)+2(k+1))/2]$

$= -[(k+1)(k+2)/2]$

$\therefore$ P(-k+1) is true.

Thus if P(-k) is true, then $P\left(\frac{1}{k}+1\right)$ is true.

By mathematical induction, P(n) is true for all nN

$\therefore (-1)+(-2)+(-3)+\ldots+(-n)= -[n(n+1)/2]$ for all $n\in$ N.

Some Finite series n

$(-1)+(-2)+(-3)+\ldots+(-n)$

$= -[1+2+3+\ldots+n]$

$= -[n(n+1)/2]$

$(-1)^2+(-2)^2+(-3)^2+\ldots+(-n)^2$

$= 1^2+2^2+3^2+\ldots+n^2$

$= n(n+1)(2n+1)/6$

$(-1)^3+(-2)^3+(-3)^3+\ldots+(-n)^3$

$= -[1^3+2^3+3^3+\ldots+n^3]$

$= -[n^2(n+1)^2/4]$

Generalization of power sums:

A reasonable generalization of the $S_n(l)$ power sums is when we sum not the power of the first positive integers, but the powers of an capricious arithmetic progression:

$S^n_{m,r}(l) = r^n+(m+r)^n+(2m+r)^n+\ldots+((l-1)m+r)$,

Where $m \neq 0$, r are co-prime integer

Obviously, $S_{1,0}{}^n(l)=S_n(l)$.

Bazso'et al proved that $S^n_{m,r}(l)$ is a polynomial of l with the explicit expression.

$$S^n_{m,r}=\frac{m}{n+1}\left(B\left(l+\frac{r}{m}\right)-B\left(\frac{r}{m}\right)\right) \quad (1)$$

Here $B_n(x)$ is still the Bernoulli polynomial, generalization of it is not necessary.

Alternating power sums:

The alternating power sums are defined as

$T_n(l) = -1^n+2^n-\ldots+(-1)^{l-1}(l-1)^n$

And can be expressed by means of the classical Euler polynomial $E_n(x)$ via:

$T_n(l) = E_n(0)+(-1)^{l-1} E_n(l)/2$,

The generalized irregular power sums are defined as

$T^n_{m,r}(l) = r^n-(m+r)^n+(2m+r)^n-\ldots+(-1)^{l-1}$

$((l-1)m+r)^n$

This can still be expressed by the Euler polynomials.

CONCLUSION

Sums of powers of negative integers have been a great interest for all the mathematicians since ages. Over many years, mathematicians in various places have given unrecorded formulas for the sum of first n negative integers. However, Thomas Harriot was the first to derive formulas for sums of powers in written form but he calculated only up to the sum of the fourth powers. Johann Faulhaber spent many months or years to derive the formula to find the sum of powers up to 17. Then, Bernoulli also spent many months or years to calculate the formula for 10^{th} power, but at a point he seemed to be hit by a pattern that needed to be compute quickly and easily, the coefficients of the formula for the sum of the c^{th} power for any negative integer c. Though, his method required one to have computed sums of lower powers, or at least they must have recorded his numbers, it was efficient enough that he accomplished the following amazing achievement.

REFERENCES

1. Boyer, C. B. "Pascal's Formula for the Sums of Powers of the Integers." *Scripta Math.* 9, 237-244.
2. Schultz, H. J. "The Sums of the k th Powers of the First n Integers." *Amer. Math. Monthly* 87, 478-481, 1980.
3. Sloane,N. J. A.Sequences A064538 and A079618 in "The On-Line Encyclopedia of Integer Sequences.
4. Yang, B.-C. "Formulae Related to Bernoulli Number and for Sums of the Same Power of Natural Numbers" [Chinese]. *Math. Pract. Th.* 24, 52-56 and 74, 1994.
5. Zhang, N.-Y. "Euler's Number and Some Sums Related to Zeta Function" [Chinese]. *Math. Pract. Th.* 20, 62-70, 1990.Referenced on Wolfram|Alpha: Power Sum.