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Stresses in an Orthotropic Elastic Layer Lying Over an Irregular Isotropic Elastic Half-Space

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ABSTRACT

Objective: The objective is to obtain the stresses due to strip loading in orthotropic plate lying over an irregular isotropic elastic medium.

Methods: Anti-plane strain problem with perfect bonding boundary conditions following by Fourier Transformation on the equilibrium equation are used to obtain the solution.

The deformation field due to shear line load at any point of the medium consisting of an orthotropic elastic layer lying over an irregular isotropic elastic medium is obtained. The anti-plane strain problem with the presence of rectangular irregularity is considered. In order to study the effect of irregularity present in the medium and of anisotropy of the layer, we computed shearing stresses in both the media graphically.

Key Words: Orthotropic, Shear load, Anti-plane strain, Rectangular irregularity

INTRODUCTION

It is well known that the upper part of the Earth is recognized as having orthorhombic symmetry. Orthorhombic Symmetry is also expected to occur in sedimentary basins as a result of combination of vertical cracks with a horizontal axis of symmetry and periodic thin layer anisotropic with a vertical symmetry axis. When one of the planes of symmetry in an orthorhombic symmetry is horizontal, the symmetry is termed as orthotropic symmetry and most symmetry systems in the Earth crust also have orthotropic orientations (Crampin[1]).

The problem of deformation of a horizontally layered elastic material due to surface loads is of great interest in geosciences and engineering. In material science engineering, the applications related to laminate composite material are increasing. Many works related to Earth, such as fills or pavements consist of layered elastic medium. When the source surface is very long, then a two-dimensional approximation simplifies the algebra and one can easily obtain a closed form analytical solution. The deformation field around mining tremors and drilling into the crust of the Earth can be analyzed by the deformation at any point of the media due to strip-loading. It also contributes for theoretical consideration of volcanic and seismic sources as it account for the deformation fields in the entire medium surrounding the source region. It may also find application in various engineering problems regarding the deformation of layered isotropic and anisotropic elastic medium (Garg *et al*[2], Singh *et al*[3]).

The study of static deformation with irregularity present in the elastic medium due to continental margin, mountain roots etc is very important to study. Chattopadhyay[4], Kar *et al*[5], De Noyer[6], Mal[7], Acharya and Roy[8] discussed the problems with irregular thickness. Love[9] provided the solution of static deformation due to line source in an isotropic elastic medium. Salim[10] studied the effect of rectangular irregularity on the static deformation of initially stressed and unstressed isotropic elastic medium respectively. The distribution of the stresses due to strip-loading in a regular monoclinic elastic medium had been studied by Madan *et al*[11]. The effect of rigidity and irregularity present in fluid-saturated porous anisotropic single layered and multilayered elastic media on the propagation of Love waves had been analyzed by Madan *et al*[12] and Kumar *et al*[13] respectively. Recently, Madan and Gabba[14] studied two-dimensional deformation of an irregular orthotropic elastic medium due to normal line load.

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In this paper, we have obtained the closed-form expressions for the displacement and shearing stresses in a horizontal orthotropic elastic plate of an infinite lateral extent lying over an irregular isotropic base due to strip-loading. Numerically, at different sizes of irregularity, we have studied the variations of shearing stresses with horizontal distance and it has been observed that the shearing stresses show significant variation with horizontal distance at the different depth levels.

PROBLEM FORMULATION

Consider a horizontal orthotropic elastic plate of thickness H lying over an infinite isotropic elastic medium with x_1–axis vertically downwards. The origin of the Cartesian coordinate system (x_1, x_2, x_3) is taken at the upper boundary of the plate. The orthotropic elastic plate occupies the region $0 \le x_1 \le H$ and is described as Medium I whereas the region $x_1 > H$ is the isotropic elastic half space over which the plate is lying and is described as Medium II. (Fig. 1)

Suppose a shear load P per unit area is acting over the strip $|x_2| \le h$ of the surface $x_1 = 0$ in the positive x_1–direction. The boundary condition at the surface $x_1 = 0$ is

$$\tau_{31} = \begin{cases} -P \; |x_2| \le h \\ 0 \; |x_2| > h \end{cases} \tag{1}$$

The irregularity is assumed to be rectangular with length $2a$ and depth d. The equation of the rectangular irregularity may be represented as

$$x_1 = \varepsilon f(x_2) = \begin{cases} d \; |x_2| \le a \\ 0 \; |x_2| > a \end{cases} \tag{2}$$

where $\varepsilon = \frac{d}{2a} << 1$ is the perturbation factor.

THEORY

The equilibrium equations of Cartesian coordinate system (x_1, x_2, x_3) for zero body force are

$$\sigma_{1,1} + \tau_{12,2} + \tau_{13,3} = 0 \tag{3}$$

$$\tau_{21,1} + \sigma_{2,2} + \tau_{23,3} = 0 \tag{4}$$

$$\tau_{31,1} + \tau_{32,3} + \sigma_{3,3} = 0 \tag{5}$$

where $\sigma_1, \sigma_2, \sigma_3$ are normal stresses and $\tau_{12}, \tau_{13}, \tau_{21}, \tau_{23}, \tau_{31}, \tau_{32}$ are called shearing stresses.

The stress-strain relations for- an orthotropic elastic medium with co-ordinate planes as planes of elastic symmetry are

$$\left.\begin{aligned} \sigma_1 &= C_{11}e_1 + C_{12}e_2 + C_{13}e_3 \\ \sigma_2 &= C_{21}e_1 + C_{22}e_2 + C_{23}e_3 \\ \sigma_3 &= C_{13}e_1 + C_{23}e_2 + C_{33}e_3 \\ \tau_{23} &= 2C_{44}e_{23} \\ \tau_{13} &= 2C_{55}e_{13} \\ \tau_{12} &= 2C_{66}e_{12} \end{aligned}\right] \tag{6}$$

where e_1, e_2, e_3 are normal strain components and e_{12}, e_{23}, e_{13} are normal strain components. The suffices $C_{ij}\,(i, j = 1,2,3,4,5,6)$ are stiffnesses of an orthotropic elastic material.

The strain - displacement relations are given as

$$e_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) \quad \text{and} \quad e_1 = \frac{\partial u_1}{\partial x_1}, \text{ etc.} \tag{7}$$

In terms of displacement components, the equilibrium equations can be written from equations (3) – (7) as :

$$C_{66}\frac{\partial^2 u_1}{\partial x_2^2} + C_{55}\frac{\partial^2 u_1}{\partial x_3^2} + (C_{12} + C_{66})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + (C_{13} + C_{55})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} = 0 \tag{8}$$

$$_6 + C_{12})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66}\frac{\partial^2 u_2}{\partial x_1^2} + C_{22}\frac{\partial^2 u_2}{\partial x_2^2} + C_{44}\frac{\partial^2 u_2}{\partial x_3^2}(C_{23} + C_{44})\frac{\partial^2 u_3}{\partial x_3 \partial x_2} = 0 \tag{9}$$

$$\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + (C_{44} + C_{23})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + C_{55}\frac{\partial^2 u_3}{\partial x_1^2} + C_{44}\frac{\partial^2 u_3}{\partial x_2^2} + C_{33}\frac{\partial^2 u_3}{\partial x_3^2} = 0 \tag{10}$$

Consider the field equation of an orthotropic material in anti – plane strain equilibrium state as:

$$u_1 = u_2 = 0, \quad u_3 = u_3(x_1, x_2); \tag{11}$$

The non-zero stresses for an anti – plane strain equilibrium state are

$$\tau_{31} = C_{55}\frac{\partial u_3}{\partial x_1} \tag{12}$$

$$\tau_{32} = C_{44}\frac{\partial u_3}{\partial x_2} \tag{13}$$

Equilibrium Equations for an orthotropic elastic medium due to anti – plane strain deformation are found to be

$$\frac{\partial^2 u_3}{\partial x_1^2} + m^2\frac{\partial^2 u_3}{\partial x_2^2} = 0 \tag{14}$$

where $m = \sqrt{\frac{C_{44}}{C_{55}}}$.

At the interface $(y, x = \varepsilon f(y))$, the boundary conditions are:

1. $u_3^I = u_3^{II}$.
2. $\tau_{31}^I - i\varepsilon f'(y)\tau_{32}^I = \tau_{31}^{II} - i\varepsilon f'(y)\tau_{32}^{II}$.

(15)

By using the boundary condition (15), we find the deformation field at any point of the orthotropic elastic plate corresponding to irregular contact between the orthotropic plate and the isotropic elastic half space due to strip-loading.

Taking the Fourier transform of the equilibrium equation (14), we get

$$\frac{d^2\bar{u}_3^I}{dx_1^2} - 2\left(\frac{C_{45}}{C_{55}} ik\right)\frac{d\bar{u}_3^I}{dx_1} - \frac{C_{44}}{C_{55}} k^2\bar{u}_3^I = 0 \qquad (16)$$

The solution of the ordinary differential equation is

$$\bar{u}_3^I = (C_1 e^{m|k|x_1} + C_2 e^{-m|k|x_1}) \qquad (17)$$

where C_1 and C_2 may be functions of k

By using inverse Fourier transform, we have

$$u_3^I = \frac{1}{2\pi}\int_{-\infty}^{\infty}(C_1 e^{m|k|x_1} + C_2 e^{-m|k|x_1})e^{-ix_2 k}dk \qquad (18)$$

Using equation (12), (13) and (18), the shear stresses are

$$\tau_{31}^I = \frac{T_1}{2\pi}\int_{-\infty}^{\infty}(C_1 e^{m|k|x_1} - C_2 e^{-m|k|x_1})e^{-ix_2 k}|k|dk \qquad (19)$$

$$\tau_{32}^I = \frac{T_1}{2\pi}\left[-im\int_{-\infty}^{\infty}(C_1 e^{m|k|x_1} + C_2 e^{-m|k|x_1})e^{-ix_2 k}kdk\right] \qquad (20)$$

Where $T_1 = mC_{55} = \sqrt{C_{44}C_{55}}$. U. Using the boundary condition (1), we have

$$\bar{\tau}_{31}^I = -\frac{2P}{\pi}\sin kh \qquad (21)$$

Therefore

$$\tau_{31}^I = -\frac{P}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin kh}{k}\right)e^{-ikx_2}dk \qquad (22)$$

From (19) and (21), we obtain

$$C_1 - C_2 = -\frac{2P}{T_1}\left(\frac{\sin kh}{k|k|}\right) \qquad (23)$$

The displacement in the isotropic elastic half space $x_1 > H$ is

$$u_3^{II} = \frac{1}{2\pi}\int_{-\infty}^{\infty} C_2' e^{-|k|x_1}e^{-ix_2 k}dk \qquad (24)$$

The coefficient C_2' is to be determined from the boundary conditions.

From equations (12), (13) and (17), we obtain

$$\tau_{31}^{II} = -\frac{\mu}{2\pi}\int_{-\infty}^{\infty} C_2' e^{-|k|x_1}e^{-ix_2 k}|k|dk \qquad (25)$$

$$\tau_{32}^{II} = -\frac{i\mu}{2\pi}\int_{-\infty}^{\infty} C_2' e^{-|k|x_1}e^{-ix_2 k}kdk \qquad (26)$$

Equations (15), (18), (19), (20), (24), (25) and (26) yield the relation

$$C_1 e^{\varepsilon m|k|f(y)} + C_2 e^{-\varepsilon m|k|f(y)} = C_2' e^{-\varepsilon|k|f(y)} \qquad (27)$$

$$T\left(k' - m\varepsilon f'(y)\right)C_1 e^{\varepsilon m|k|f(y)} - T\left(k' + m\varepsilon f'(y)\right)C_2 e^{-\varepsilon m|k|f(y)} + \left(k' + \varepsilon f'(y)\right)C_2' e^{-\varepsilon|k|f(y)} \qquad (28)$$

where $T = \frac{T_1}{\mu}$ and $k' = \frac{|k|}{k}$.

Solving (23), (27) and (28), we get

$$C_1 = \frac{2P\sin kh}{T_1 k|k|}\left(\frac{(k'+\varepsilon f'(y)V')e^{-2\varepsilon m|k|f(y)})}{k'(V-e^{-2\varepsilon m|k|f(y)})-\varepsilon f'(y)V'(1+e^{-2\varepsilon m|k|f(y)})}\right) \qquad (29)$$

$$C_2 = \frac{2P\sin kh}{T_1 k|k|}\left(1 + \frac{(k'+\varepsilon f'(y)V')e^{-2\varepsilon m|k|f(y)})}{k'(V-e^{-2\varepsilon m|k|f(y)})-\varepsilon f'(y)V'(1+e^{-2\varepsilon m|k|f(y)})}\right) \qquad (30)$$

$$C_2' = \frac{2P\sin kh}{T_1 k|k|}\left(\frac{k'(1+V)}{k'(V-e^{-2\varepsilon m|k|f(y)})-\varepsilon f'(y)V'(1+e^{-2\varepsilon m|k|f(y)})}\right)e^{-\varepsilon(m-1)|k|f(y)} \qquad (31)$$

where $V = (T-1)/(T+1)$.

PROBLEM SOLUTION

By applying Fourier Transformation technique on equation (2) we obtained

$$\bar{f}(k) = \frac{4a}{k}\sin(ka) \qquad (32)$$

Therefore, by using inverse transformation, we have

$$f(y) = \frac{1}{2\pi}\sin(ka)\, e^{-iky}\, dk = sign(a-x_2) + sign(a+x_2) \qquad (33)$$

where is the signum function.

By substituting the values of constants C_1,C_2 and C_1' from equations (29), (30), (31) in the equations (18), (19), (20) for Medium I and in (24), (25), (26) for Medium II and also, substituting the value of *f(y)* for rectangular irregularity from equation (33), we obtain the following expressions for displacement and stresses.

For Med. I

$$u_3^I = \frac{P}{\pi T_1}\int_{-\infty}^{\infty}\frac{\sin kh}{k|k|}\left\{\left(1+\sum_{n=1}^{\infty}V^n e^{m|k|\left(2n\varepsilon(sign(a-x_2)+sign(a+x_2))\right)}\right)\left(e^{m|k|x_1} + Ve^{-m|k|x_1}\right)\right\}e^{-ikx_2} \qquad (34)$$

$$\tau_{31}^I = \frac{P}{\pi}\left[(1+V)\tan^{-1}\left(\frac{2hmx_1}{x_2^2+m^2x_1^2-h^2}\right) + \sum_{n=1}^{\infty}V^n\left\{\tan^{-1}\left(\frac{2hm\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)}{x_2^2+m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)^2-h^2}\right) - V\tan^{-1}\left(\frac{2hm\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)}{x_2^2+m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2-h^2}\right)\right\}\right] \qquad (35)$$

$$\tau_{32}^I = -\frac{Pm}{4\pi}\left[(1+V)\log\frac{(h+x_2)^2+m^2x_1^2}{(h-x_2)^2+m^2x_1^2} - \sum_{n=1}^{\infty}V^n\left\{\log\frac{(h+x_2)^2+m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)^2}{(h-x_2)^2+m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)+x_1\right)^2} + V\log\frac{(h+x_2)^2+m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}{(h-x_2)^2+m^2\left(2n\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}\right\}\right] \qquad (36)$$

For Med. II

$$u_3^{II} = -\frac{P}{\pi T_1}\int_{-\infty}^{\infty}\frac{\sin kh}{k|k|}(1+V)\left(1 + \sum_{n=1}^{\infty} V^n e^{2mn\varepsilon\,|k|\left(sign\,(a-x_2)+sign\,(a+x_2)\right)}\right) e^{|k|\left((m+1)\varepsilon\left(sign\,(a-x_2)+sign\,(a+x_2)\right)-x_1\right)} e^{-ikx_2}dk \qquad (37)$$

$$\tau_{31}^{II} = -\frac{P\mu}{\pi T_1}(1+V)\left[\tan^{-1}\left(\frac{2h\left((m+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)}{x_2^2+\left((m+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2-h^2}\right) + \sum_{n=1}^{\infty} V^n\left\{\tan^{-1}\left(\frac{2h\left((2m(n+1)+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)}{x_2^2+\left((2m(n+1)+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2-h^2}\right)\right\}\right] \qquad (38)$$

$$\tau_{32}^{II} = \frac{P\mu}{2\pi T_1}(1+V)\left[\log\frac{(h+x_2)^2+\left((m+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}{(h-x_2)^2+\left((m+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2} + \sum_{n=1}^{\infty} V^n \log\frac{(h+x_2)^2+\left((2m(n+1)+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}{(h-x_2)^2+\left((2m(n+1)+1)\varepsilon\left(sign(a-x_2)+sign(a+x_2)\right)-x_1\right)^2}\right] \qquad (39)$$

NUMERICAL RESULTS AND DISCUSSION

In this section, we intend to examine the effect of irregularity on the stresses due to shear line load acting at any point of the orthotropic elastic layer lying over an irregular isotropic half space. For numerical computation, we use the values of elastic constants of Topaz (Orthotropic) for Medium I and the values of elastic constants of Glass (Isotropic) for Medium II given by Love[9].

Figures (2)-(4) and Figures (5)-(7) show the variation of shearing stresses τ_{31}^{I} and τ_{32}^{I} respectively, with horizontal distancefor different values of $a = 1, 1.2, 1.4, 1.6$ and for different depth levels $x_1 = 0.5, 1, 1.5$. Figures (5)-(7) clearly show that for different values of, the difference between shearing stresses in magnitude significantly decreases as the depth increases.

Figures (8)-(10) and Figures (11)-(13) show the variation of shearing τ_{31}^{II} and τ_{32}^{II} respectively with horizontal distancefor x_2 different values of $a = 1, 1.2, 1.4, 1.6$. It has been found from the Figures (8)-(10) that for different values of *a*, the difference between shearing stresses in τ_{31}^{II} magnitude significantly increases as the depth increases.

CONCLUSIONS

The explicit expressions for the shearing stresses in an elastic medium consisting of orthotropic elastic layer lying over an irregular isotropic half space due to shear loading has been obtained. The results obtained are useful to study the static deformation around mining tremors and drilling into the crust of the Earth. The results are also useful to study the effect of irregularity present between the layer and the half-space. Graphically, it has been observed that the difference between the shearing stresses in magnitude in orthotropic elastic layer decreases as depth increases due to irregularity present.

Further, it has also been observed that in isotropic semi-infinite half-space, the difference between the stresses in magnitude increases with the increase of depth. Thus, it has been concluded that the stress distribution in a layer with irregularity present at the interface is affected by not only the presence of irregularity but also by anisotropy of the elastic medium as a result of shear load acting over the strip of an orthotropic elastic medium.

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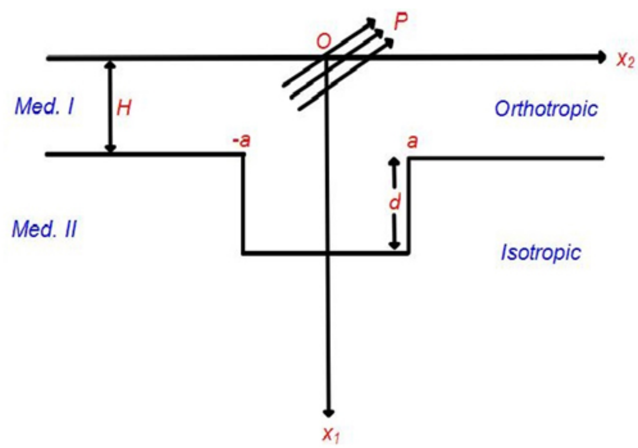

Figure 1: Section of the Model.

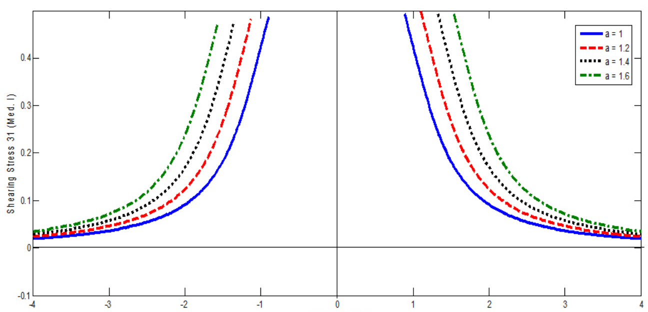

Figure 2: Variation of the Shearing Stress τ_{31}^{I} in Med. I with the horizontal distance x_2 at x_1= 1.

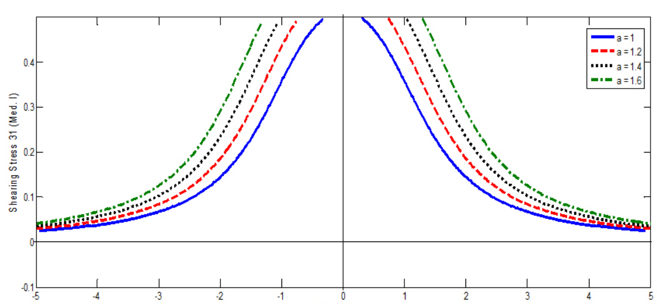

Figure 3: Variation of the Shearing Stress τ_{31}^{I} in Med. I with the horizontal distance x_2 at x_1= 1.

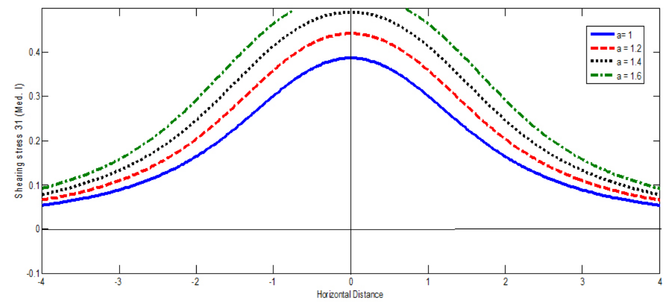

Figure 4: Variation of the Shearing Stress τ_{31}^{I} in Med. I with the horizontal distance x_2 at x_1= 1.5.

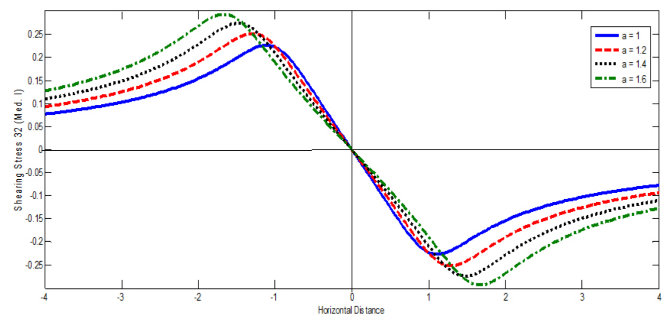

Figure 5: Variation of the Shearing Stress τ_{32}^{I} in Med. I with the horizontal distance x_2 at x_1= 0.5.

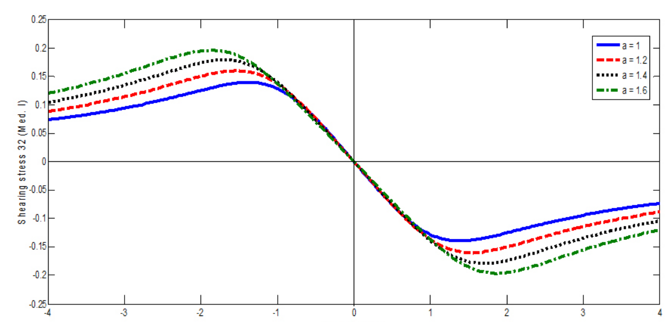

Figure 6: Variation of the Shearing Stress τ_{32}^{I} in Med. I with the horizontal distance x_2 at x_1= 1.

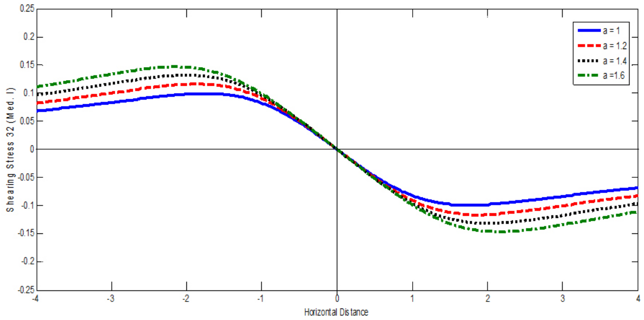

Figure 7: Variation of the Shearing Stress τ_{32}^{I} in Med. I with the horizontal distance x_2 at x_1= 1.5.

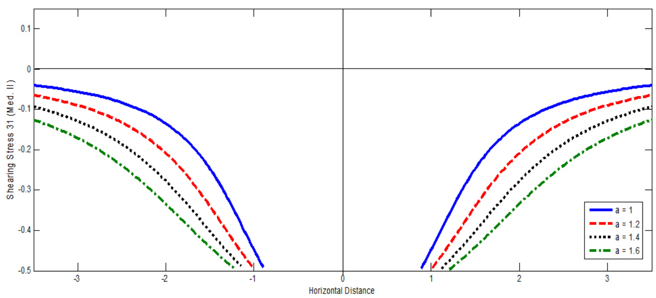

Figure 8: Variation of the Shearing Stress τ_{31}^{II} in Med. II with the horizontal distance x_2 at x_1= 0.5.

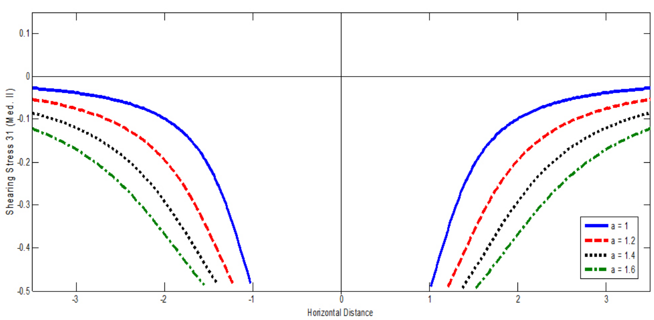

Figure 9: Variation of the Shearing Stress τ_{31}^{II} in Med. II with the horizontal distance x_2 at x_1= 1.

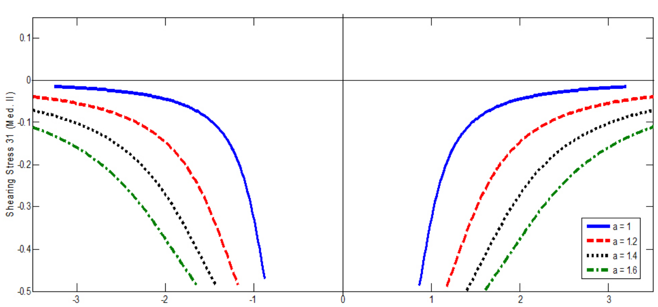

Figure 10: Variation of the Shearing Stress τ_{31}^{II} in Med. II with the horizontal distance x_2 at x_1= 0.5.

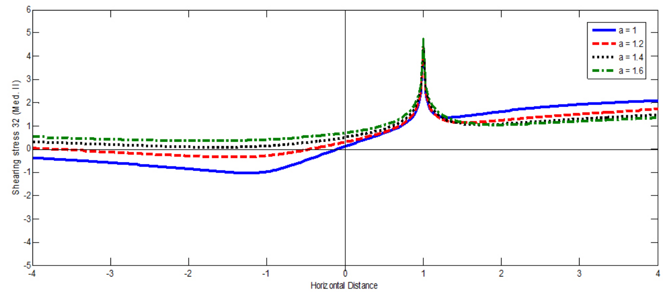

Figure 11: Variation of the Shearing Stress τ_{32}^{II} in Med. II with the horizontal distance x_2 at x_1= 0.5.

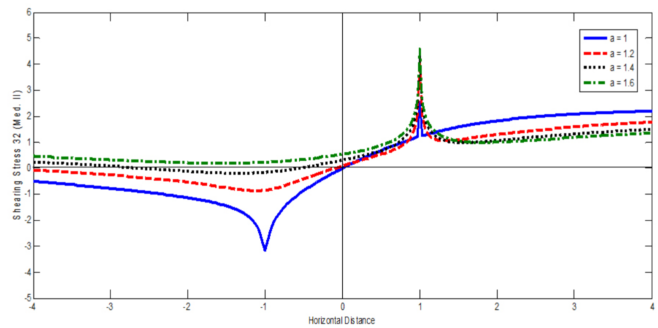

Figure 12: Variation of the Shearing Stress τ_{32}^{II} in Med. II with the horizontal distance x_2 at x_1= 1.

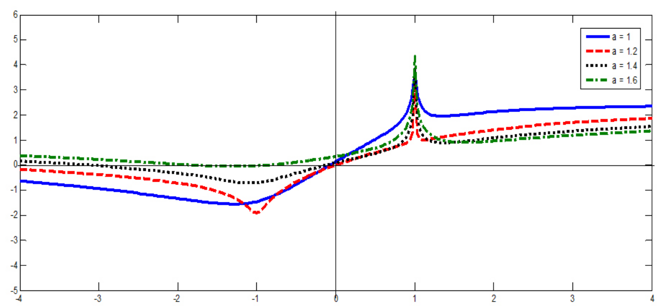

Figure 13: Variation of the Shearing Stress τ_{32}^{II} in Med. II with the horizontal distance x_2 at x_1 = 1.5.